Total No. of Questions :5]

SEAT No. :

P2436

[Total No. of Pages :3

[4939]-41
M.Sc.Tech.
MATHEMATICS
Industrial Mathematics with Computer Applications
MIM-401: Topology
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any <u>eight</u> of the following: **[16]**

a) Find all open and closed subsets of $X = \{a, b, c\}$, where topology on X is $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$.

b) Is the collection $J_\infty = \{U \mid X - U$ is infinite or all of $X\}$ a topology on X?

c) Show that (0, 1) and [0, 1] with usual topology are not homeomorphic.

d) State Tychonoff's theorem.

e) Let X be a Hausdorff space and $x \in X$. Show that $\{x\}$ is a closed set in X.

f) Define first countable space.

g) Give an example of an non-Hausdorff space. Justify.

h) State Urysohn's Lemma.

i) Show that $\mathbb{R}$, with usual topology is locally compact.

j) Define Lindelöf space.

P.T.O.

Q2) a) Attempt any <u>one</u> of the following: **[6]**

i) If $\{\tau_\alpha\}$ is a collection of topologics on X, then show that $\bigcap_\alpha \tau_\alpha$ is a topology on X. Is $\bigcup_\alpha \tau_\alpha$ a topology on X? Justify your answer.

ii) Show that subspace and product of a Hausdorff space is Hausdorff.

b) Attempt any <u>two</u> of the following: **[10]**

i) Let A, B denote subsets of a topological space X, then prove the following

1) If $A \subset B$, then $\bar{A} \subset \bar{B},$.

2) $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

ii) If X is a Hausdorff space, then a sequence of points of X converges to atmost one point of X.

iii) Show that a topological space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x \mid x \in X\}$ is closed in X ×X.

Q3) a) Attempt any <u>one</u> of the following: **[6]**

i) Prove that every T_3-space is a T_2-space.

ii) Let $F: A \to XXY$ be given by the equation $f(a) = (f_1(a), f_2(a)), \forall a \in A$. Prove that f is continuous if and only if the functions $f_1 : A \to X, f_2 : A \to Y$ are continuous.

b) Attempt any <u>two</u> of the following: **[10]**

i) Show by an example that intersection of two compact spaces need not be compact.

ii) Show that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 3x + 1$ is a homomorphism.

iii) Prove that the image of a connected space under a continuous map is connected.

Q4) a) Attempt any one of the following: **[6]**

i) State and prove intermediate value theorem.

ii) Show that every compact subspace of a Hausdorff space is closed.

b) Attempt any two of the following: **[10]**

i) Prove that a closed subspace of a Lindelöf space is Lindelöf.

ii) Show that the space $\mathbb{R}_l$ is normal.

iii) Suppose that X has a countable basis. Then prove that every open covering of X contains a countable subcollection covering X.

Q5) a) Attempt any one of the following: **[6]**

i) Prove that every second countable space is first countable. Give an example of a first countable space X which is not second countable.

ii) Prove that every metrizable space is normal.

b) Attempt any two of the following: **[10]**

i) Prove that one point compactification of $\mathbb{R}$ is S^1.

ii) Give an example of a topological space X which is limit point compact but not compact.

iii) Prove that every second countable space is separable.

ℰ ℰ ℰ