Total No. of Questions :8]

SEAT No. :

P2700

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[5039]-203

M.Sc.

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM-203: Numerical analysis

(2013 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) ***Attempt any five of the following.***
2) ***Figures to the right indicate full marks.***
3) ***Use of non-programmable scientific calculator is allowed.***

Q1) Attempt the following:

a) Use false position method to determine the roots of the equation $e^{-x} - x = 0$. Two initial guess values being $x_0 = 0$ and $x_1 = 1$.

Compute first TWO iterations. **[4]**

b) Start with $f(x) = x^3 - A$, where A is any real number, and determine recursive formula $P_k = \dfrac{2P_{k-1} + \dfrac{A}{P_{k-1}^2}}{3}$, for k = 1, 2, ... **[4]**

c) Find a root of equation '$x^3 - x - 4 = 0$' using bisection method which lies in [1, 2] correct upto 2 - places of decimal. **[2]**

Q2) Attempt the following:

a) Obtain the Newton - Raphson formula to find the root of the equation $f(x) = 0$. **[4]**

b) Construct the difference table from the following data to obtain $f(50.5)$; $f(50) = 39.1961$, $f(51) = 39.7981$, $f(52) = 40.3942$, $f(53) = 40.9843$ $f(54) = 41.5687$ **[4]**

c) Discuss ill conditioned system. **[2]**

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Q3) Attempt the following:

a) Assume that $f \in C^3[a,b]$ and that $x-h,\ x,\ x+h \in [a,b]$. Prove that

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}.$$ **[4]**

b) State and prove 'Composite Trapezoidal Rule'. **[4]**

c) Find fixed point if any of $g(x) = -4 + 4x - \frac{x^2}{2}$. **[2]**

Q4) Attempt the following:

a) Use numerical differentiation formula

$f''(x) = \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$ to approximate '$f(x) = \cos(x)$' at x = 0.8, with h = 0.01. Compare your result with the true value of $f''(0.8)$. **[4]**

b) From the following data, find $\sqrt{1.1}$ using Lagrange's interpolation. Determine the accuracy of interpolation. **[4]**

x	1	1.2	1.3	1.4
$\sqrt{x}$	1	1.095	1.140	1.183

c) Find the absolute error and relative error in the approximation of **[2]**

x = 2.71 8 2 8 1 8 2 by

$\bar{x}$ = 2.71 8 2

Q5) Attempt the following:

a) Find the parabola $y = A + Bx + Cx^2$ that passes through the three points (1, 1), (2, –1), (3, 1). **[4]**

b) Find the Jacobian matrix J(x, y, z) of order 3 × 3 at the point (1, 3, 3) for the three functions. **[4]**

$$f_1(x,y,z) = x^3 - y^3 - y - z^4 + z^2$$

$$f_2(x,y,z) = xy + yz + xz$$

$$f_3(x,y,z) = \frac{y}{xz}$$

c) State Simpson's $\frac{3}{8}^{\text{th}}$ rule. **[2]**

Q6) Attempt the following:

a) Using Euler's method, obtain the solution of $y' = x - y$, given: $x_0 = 0$, $y_0 = 1$ at $x = 0.6$ by taking step size $h = 0.2$. **[4]**

b) Solve the following system of linear equations using Gauss seidel iterative method. [Perform 2 - iterations]. **[4]**

$$9x_1 + 2x_2 + 4x_3 = 20$$
$$2x_1 - 4x_2 + 10x_3 = -15$$
$$x_1 + 10x_2 + 4x_3 = 6$$

c) Let λ, ν be an eigen pair of a matrix A. If α is any constant, show that $\lambda - \alpha$, ν is an eigen pair of matrix $A - \alpha I$. **[2]**

Q7) Attempt the following:

a) Derive Newton's Forward difference formula. **[5]**

b) Using Modified Euler's method, solve the following differential equation,

$y' = 1 + xy$ with y = 1 at $x = 0$. Find value of y at $x = 0.1$. **[5]**

Q8) Attempt the following:

a) Use Runge - Kutta method of fourth order to solve the initial value problem $y' = x + y$ when y = 1 at $x = 0$.

Find solution for $x = 0.1$. **[5]**

b) Solve the following system of linear equations using L-U decomposition. **[5]**

$$3x + 2y + 4z = 7$$
$$2x + y + z = 7$$
$$x + 3y + 5z = 2$$

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