Total No. of Questions :8]

SEAT No. :

P2699

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[5039]-202

M.Sc. (IMCA)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

Mathematics

MIM-202: Algebra- I

(2013 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define order of an element in a group G. **[2]**

b) State and prove Lagrange theorem for finite groups. **[4]**

c) Let G be a group and H a non empty subset of G. Prove that H is a subgroup of G if ab^{-1} is in H whenever a & b are in H. **[4]**

Q2) a) Is union of two subgroups a subgroup? Justify. **[2]**

b) Let $G = \langle a \rangle$ be a cyclic group of order n. Prove that $G = \langle a^k \rangle$ if and only if $gcd\ (k, n) = 1$. **[4]**

c) Prove that if H has index 2 in G, then H is normal in G. **[4]**

Q3) a) Give an example of a non cyclic group, all of whose proper subgroups are cyclic. **[2]**

b) Let $\langle a \rangle = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there? List them. **[4]**

c) Define transposition. Also prove that A_n is a subgroup of S_n. **[4]**

Q4) a) State and prove Cayley's theorem. **[5]**

b) Let G be a group and let Z(G) be the center of G. If G/Z(G) is cyclic, then prove that G is abelian. **[5]**

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Q5) a) State and prove the First Isomorphism theorem for rings. **[5]**

b) Prove that R/A is a field if and only if A is maximal. **[5]**

Q6) a) Prove that the characteristic of an integral domain is O or a prime. **[4]**

b) Show that every non zero element of $\mathbb{Z}_n$ is a unit or a zero divisor. **[4]**

c) Define simple group. **[2]**

Q7) a) Define prime ideal of a ring R. **[2]**

b) Let $\mathbb{Z}\left[\sqrt{2}\right] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$. Prove that $\mathbb{Z}\left[\sqrt{2}\right]$ is a ring under the ordinary addition and multiplication of real numbers. **[4]**

c) Prove that a finite integral domain is a field. **[4]**

Q8) a) Define divisors of zero in a ring R. **[2]**

b) Prove that if D is an integral domain, then D[x] is an integral domain. **[4]**

c) For any prime P, prove that the Pth cyclotomic polynomial $\phi_p(x) = x^{p-1} + x^{p-2} + \text{---} + x + 1$ is irreducible over $\mathbb{Q}$. **[4]**

EEE