Total No. of Questions :8]

SEAT No. :

P2655

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M.Sc.

PHYSICS

PHY UT 503: Mathematical Methods in Physics

(2013 Pattern) (5- Credits) (Credit System) (Semester - I)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Answer ANY FIVE questions out of eight questions.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Use of calculator is allowed.*

Q1) a) Define Basis and Dimension of a Vector Space. Explain with one example. **[4]**

b) Evaluate $\oint_c \frac{e^z}{z(z+1)} dz$ where C is the circle $|z - 1| = 3$. **[3]**

c) Obtain the Associated Legendre function $P_2^3(x)$. **[3]**

Q2) a) State and prove a necessary condition (Cauchy - Riemann Equations) for a function $w = f(z) = u(x, y) + iv(x, y)$ to be analytic in a region R. **[4]**

b) Find the Fourier coefficients a_n and b_n in the interval (–L, +L) for odd function. **[3]**

c) Determine whether or not the following vectors in R^3 are linearly dependent: $\{(1, 0, 0), (0, 1, 0), (0, 0, 0)\}$. **[3]**

Q3) a) State and prove Cauchy-Schwarz inequality. **[4]**

b) For which value of K will the vector u = (1, –2, K) in R^3 be a linear combination of the vectors V = (3, 0, –2) and W = (2, –1, –5)? **[3]**

c) Determine the region in the z - plane represented by $1 < |z + 2i| \leq 2$. **[3]**

P.T.O.

Q4) a) Consider the following basis of Euclidean space R^3: **[4]**

$$\{V_1 = (1,1,1),\ V_2 = (0,1,1),\ V_3 = (0,0,1)\}$$

By using Gram schmidt orthogonalization process to transform $\{V_i\}$ into an orthonormal basis $\{u_i\}$

b) Let $V = R^3$. Show that W is a subspace of V, where $W = \{(a,b,c)/\ a+b+c=0\}$. **[3]**

c) Prove that: $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$. **[3]**

Q5) a) Determine the first three Laguerre polynomials $L_0(x)$, $L_1(x)$ and $L_2(x)$. **[4]**

b) Prove that the Inverse Laplace transform operator L^{-1} is linear. **[3]**

c) Determine the residue of $\dfrac{z^2}{(z-2)(z^2+1)}$ at $z = i$. **[3]**

Q6) a) Prove that if $\mathscr{L}\{f(t)\} = F(S)$ then **[4]**

$$\mathscr{L}\{f(at)\} = \frac{1}{a} F\left(\frac{S}{a}\right)$$

b) Prove that: **[3]**

$$H_{n+1}(x) = 2x\,H_n(x) - 2n\,H_{n-1}(x).$$

c) Discuss whether or not R^2 is a subspace of R^3. **[3]**

Q7) a) Let T be the linear operator on R^3 defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. **[5]**

i) Find the matrix of T in the basis $\{f_1 = (1,1,1),\ f_2 = (1,1,0),\ f_3 = (1,0,0)\}$

ii) Verify that $[T]_f\,[V]_f = [T(V)]_f$ for any vector $V \in R^3$.

b) State and prove the orthogonality property of Legendre polynomials. **[5]**

Q8) a) Find $\mathcal{L}^{-1}\left\{\dfrac{3s+1}{(s-1)(s^2+1)}\right\}$ **[5]**

b) Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$. **[5]**

Find a (real) orthogonal matrix P for which $P^{-1}AP$ is diagonal.