Total No. of Questions : 8]

SEAT No. :

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[4922]-103
M.Sc.
PHYSICS
PHY UT-503 : Mathematical Methods in Physics
(2013 Pattern - 5 Credits) (Semester-I) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Answer any five questions out of eight questions.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Use of calculator is allowed.***

Q1) a) Define vector space and subspace. Discuss whether or not R^3 is a subspace of R^4. **[4]**

b) Determine the residue of $\frac{ze^{zt}}{(z-3)^2}$ at $z = 3$. **[3]**

c) Obtain the first three Legendre polynomials using Rodrigue's formula. **[3]**

Q2) a) State Residue theorem. Explain how the Cauchy's theorem and integral formulas are special cases of residue theorem. **[4]**

b) State and prove the Parseval's identity. **[3]**

c) Let V = R^3. Determine whether or not W is a subspace of V.

Given: W = $\left\{(a, b, c) : a^2 + b^2 + c^2 \leq 1\right\}$. **[3]**

Q3) a) State and prove Cauchy-Schwarz inequality. **[4]**

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b) Determine whether or not the following form a basis for the vector space R^3: {(1, 2, 3), (1, 0, –1), (3, –1, 0), (2, 1, –2)}. **[3]**

c) Determine the region in the z plane represented by $\pi/3 \le arg(z) \le \pi/2$. **[3]**

Q4) a) Verify that the following is an inner product in R^2:

$$< u, v >= x_1\ y_1 - x_1 y_2 - x_2 y_1 + 3x_2 y_2$$

where $u = (x_1, x_2), v = (y_1, y_2)$. **[4]**

b) For what value of k is (1, k, 5) a linear combination of u = (1, –3, 2) and v = (2, –1, 1). **[3]**

c) Prove that: $J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x).$ **[3]**

Q5) a) Determine the first three Hermite polynomials $H_0(x)$, $H_1(x)$ and $H_2(x)$. **[4]**

b) Prove that the Laplace transform operator L is linear. **[3]**

c) Evaluate $\oint_C \frac{\cos z}{(z-\pi)} dz$ where C is the circle $|z-1| = 3$. **[3]**

Q6) a) Let $f(t)$ be continuous and have a piecewise. Continuous derivative $f'(t)$ in every finite interval $0 \le t \le T$. Suppose also that $f(t)$ is of exponential order for $t > T$. Then prove that $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$. **[4]**

b) Prove that: $H_{n+1}(x) = 2xH_n(x) - 2_n H_{n-1}(x)$. **[3]**

c) Write the vector v = (3, 1, –4) as a linear combination of $f_1 = (1,1,1)$, $f_2 = (0,1,1)$ and $f_3 = (0,0,1)$. **[3]**

Q7) a) Find $\mathcal{L}^{-1}\left\{\dfrac{5s^2-15s+7}{(s+1)(s-2)^3}\right\}$. **[5]**

b) Consider the following basis of Euclidean space R^2: $\{v_1 = (1, 1, 1), v_2 = (0, 1, 1), v_3 = (0, 0, 1)\}$. Use the Gram-Schmidt orthogonalization process to transform $\{v_i\}$ into an orthonormal basis $\{u_i\}$. **[5]**

Q8) a) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and let T be the linear operator on R^2 defined by $T(v) = Av$ (where v is written as a column vector). Find the matrix of T in each of the following bases: **[5]**

i) $\{e_1 = (1, 0), e_2 = (0, 1)\}$, i.e. usual basis;

ii) $\{f_1 = (1, 3), f_2 = (2, 5)\}$.

b) State and prove the orthogonality property of Hermite functions. **[5]**

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