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SEAT No. :

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**P2693**

**[5039]-101**

**M.Sc.**

**INDUSTRIAL MATHEMATICS WITH COMPUTER  
APPLICATIONS**

**MIM-101 : Real Analysis  
(2013 Pattern) (Semester-I)**

*Time : 3 Hours]*

*[Max. Marks : 50*

*Instructions to the candidates:*

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicate full marks.*
- 3) *Unless specified,  $\mathbb{R}^n$  is assumed to have usual metric for all  $n \geq 1$ .*

**Q1)** a) Prove that a set  $E$  of a metric space is open if and only if its complement is closed. **[4]**

b) If  $F$  is a closed subset and  $K$  is a compact subset of a metric space  $M$  then prove that  $F \cap K$  is compact. **[3]**

c) Let  $\{P_n\}_{n=1}^{\infty}$  be a sequence in a metric space  $X$ . Prove that if  $\{P_n\}$  is convergent then it is bounded. **[3]**

**Q2)** a) If  $\{P_n\}$  is a sequence in a compact metric space  $X$ , then prove that some subsequence of  $\{P_n\}$  converges to a point of  $X$ . **[4]**

b) Prove that the convergence of  $\{S_n\}$  implies convergence of  $\{S_n|\}$ . Is converse true? **[3]**

c) If  $P > 0$  then prove that  $\lim_{n \rightarrow \infty} \frac{1}{n^P} = 0$ . **[3]**

**P.T.O.**

- Q3)** a) Prove that closed subset of a compact set is closed. [4]
- b) If  $X$  is a metric space and  $E \subseteq X$ , then prove that  $E = \bar{E}$  if and only if  $E$  is closed. [4]
- c) Construct a bounded set of real numbers with exactly three limit points. [2]
- Q4)** a) If  $S_1 = \sqrt{2}$  and  $S_{n+1} = \sqrt{2 + \sqrt{S_n}}$ ,  $n = 1, 2, 3, \dots$  then prove that  $\{S_n\}$  is convergent. [4]
- b) Prove that the series  $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$  is convergent if  $p > 1$ . [4]
- c) Find the radius of convergence of  $\sum n^n z^n$ , where  $z$  is a complex number. [2]
- Q5)** a) If  $f$  is a continuous mapping of a metric space  $X$  into a metric space  $Y$  and  $E$  is a connected subset of  $X$  then prove that  $f(E)$  is connected. [5]
- b) Let  $f(x) = |x|^3$ . Compute  $f'(0)$  if it exists. [3]
- c) Let  $f$  be defined on  $[a, b]$ . If  $f$  is differentiable at a point  $x \in [a, b]$  then prove that  $f$  is continuous at  $x$ . [2]
- Q6)** a) Let  $f$  be defined on  $[a, b]$ ; if  $f$  has a local maximum at a point  $x \in [a, b]$  and if  $f'(x)$  exists then prove that  $f'(x) = 0$ . [5]
- b) If  $c_0 + \frac{c_1}{2} + \dots + \frac{c_{n-1}}{n} + \frac{c_n}{n+1} = 0$  where  $c_0, c_1, \dots, c_n$  are real constants prove that the equation  $c_0 + c_1 x + c_2 x^2 + \dots + c_{n-1} x^{n-1} + c_n x^n = 0$  has at least one real root between 0 and 1. [3]

c) Let  $f$  be defined by

$$f(x) = \begin{cases} x \sin \frac{1}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

Prove that  $f'(0)$  does not exist. [2]

**Q7)** a) Suppose  $f$  be a bounded real function defined on  $[a, b]$ . Prove that  $f \in \mathbb{R}(\alpha)$  if and only if for every  $\epsilon > 0$  there exists a partition  $P$  such that  $U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$ . [5]

b) i) If  $f \in \mathbb{R}(\alpha)$  and  $g \in \mathbb{R}(\alpha)$  on  $[a, b]$  then prove that  $fg \in \mathbb{R}(\alpha)$  on  $[a, b]$  then prove that  $fg \in \mathbb{R}(\alpha)$ .

ii) Let  $f$  be defined on  $[a, b]$  as follows

$$\begin{aligned} f(x) &= 0 & \text{if } x \text{ is irrational} \\ &= 1 & \text{if } x \text{ is rational} \end{aligned}$$

Prove that  $f$  is not Riemann integrable on  $[a, b]$  [5]

**Q8)** a) Let  $f_n(x) = n^2 x(1 - x^2)^n$ ,  $0 \leq x \leq 1$ ,  $n = 1, 2, 3, \dots$ . [5]

i) Find  $\lim_{n \rightarrow \infty} f_n(x)$ .

ii) Show that  $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx \neq \int_0^1 \left[ \lim_{n \rightarrow \infty} f_n(x) \right] dx$ .

b) Suppose  $\{f_n\}$  converges  $f$  uniformly on a set  $E$  in a metric space. Let  $x$  be a limit point of  $E$  and suppose that  $\lim_{t \rightarrow x} f_n(t) = A_n$ ,  $n = 1, 2, \dots$ . Then prove that  $\{A_n\}$  is convergent and  $\lim_{t \rightarrow x} f(t) = \lim_{n \rightarrow \infty} A_n$ . [5]

