Total No. of Questions : 5]

SEAT No. :

P4089

[Total No. of Pages : 3

[5039]-12

M.Sc. Tech. (Semester - I)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 102 : Algebra - I

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicates full marks.*

Q1) Attempt any eight of the following : **[8 × 2 = 16]**

a) Let G be a group of all 2 × 2 non-singular matrices with real entries, under matrix multiplication. Show that H = $\left\{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} / a \neq 0 \text{ in } \mathbb{R}\right\}$ is normal subgroup of G.

b) Say true or false and justify your answer : "Every proper subgroup of a group of order 77 is cyclic".

c) Find the order of the permutation $\sigma = (1,2,3)(3,2)(1,2)$.

d) Define unit in a ring. Find the units in the ring of integers $\mathbb{Z}$.

e) If 'a' is generator of a cyclic group G, show that a^{-1} is also a generator.

f) Is every integral domain a field? Justify your answer.

g) If R is a ring with unity 1 and ϕ is a ring homomorphism of R onto R^1 prove that, $\phi(1)$ is the unity of R^1.

h) Give an example of a subring of a ring which is not an ideal.

i) Find characteristics of the following rings.

   i) $2\mathbb{Z}$

   ii) $\mathbb{Z}_3 \times \mathbb{Z}_4$

j) Is $x^3 + 2x + 3$ an irreducible polynomial in $\mathbb{Z}_5[x]$? Justify your answer.

P.T.O.

Q2) A) Attempt any one of the following : **[1 × 6 = 6]**

a) Prove that every subgroup of a cyclic group is cyclic.

b) State and prove the necessary and sufficient conditions for a non-empty subset H of a group G to be a subgroup.

B) Attempt any two of the following : **[2 × 5 = 10]**

a) Show that any subgroup of a group of index 2 is normal subgroup.

b) Let $\sigma = (2,3,1,4)(4,6)(2,1,5)$ be a permutation in S_6. Check whether 'σ' is even or odd permutation.

c) If G is a finite group of prime order, then show that G is cyclic group.

Q3) A) Attempt any one of the following : **[1 × 6 = 6]**

a) Let G be a group and $0(G) = P^2$, where 'p' is a prime number. Prove that G is abelian.

b) Let ϕ be a homomorphism of a group G onto $\overline{G}$ with kernel K. Prove that $\frac{G}{K} \cong \overline{G}$.

B) Attempt any two of the following : **[2 × 5 = 10]**

a) Prove that a group of order 42 is not simple.

b) Let N and M be two normal subgroups of a group G such that $N \cap M = \{e\}$ show that $n \cdot m = m \cdot n, \forall\ n \in N \ \&\ \forall\ m \in M$.

c) Define : conjugacy class. Find conjugacy classes of S_3.

Q4) A) Attempt any ONE of the following : **[1 × 6 = 6]**

a) Prove that a commutative ring R with unity is integral domain if and only if cancellation laws hold in R.

b) Let R be a commutative ring with unity and M be a maximal ideal of R, then prove that $\frac{R}{M}$ is a field.

B) Attempt any two of the following : **[2 × 5 = 10]**

a) Find all prime ideals of $\mathbb{Z}_{10}$.

b) Prove that the homomorphism ϕ of a ring R onto the ring R' is an isomorphism if and only if Kernel of ϕ is $\{0\}$.

c) Let $U = < x^2 + x + 4 >$ be an ideal of $\mathbb{Z}_{11}[x]$ generated by $x^2 + x + 4$.

Show that $\mathbb{Z}_{11}[x]/U$ is a field.

Q5) A) Attempt any one of the following : **[1 × 6 = 6]**

a) State and prove division algorithm for polynomials ring $F[x]$, where F is a field.

b) Prove that if R is integral domain then so is polynomial ring R [x].

B) Attempt any two of the following : **[2 × 5 = 10]**

a) Find all units in the ring of Gaussian integers $\mathbb{Z}[i]$. Further, if a + ib is not a unit in $\mathbb{Z}[i]$, show that $a^2 + b^2 > 1$.

b) If 'p' is a prime number, show that the polynomial $f(x) = 1 + x + x^2 + \dots\dots + x^{p-1}$ is irreducible over rationals.

c) Prove that the product of two primitive polynomials is a primitive polynomial.

❍❍❍