Total No. of Questions : 8]

SEAT No. :

P1246

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[5121]-45

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT - 805 : Lattice Theory

(2008 Pattern)

Time : 3 Hour] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let No be the set of all non-negative integers. Define $m \leq n$ if and only if there exists $k \varepsilon N_o$ such that $n = km$.

Prove that N_o is a lattice under this relation. **[6]**

b) Let L be a lattice then prove that I is a proper ideal of L if and only if there is a join-homomorphism ϕ of L onto C_2 such that $I = \phi^{-1}(0)$. **[6]**

c) Prove that every meet homomorphism is isotone. Is the converse true? Justify? **[4]**

Q2) a) Prove that every homomorphic image of a lattice L is isomorphic to a suitable quotient lattice of L. **[6]**

b) Let L be a lattice and let I be nonempty subset of L. Prove that I is an ideal if and only if a, b ε I implies that $a v b \in I$, and $a \in I$, $x \in L$, $x \leq a$ imply that $x \in I$. **[5]**

c) Let θ be a congruence relation of lattice L Then prove that for every $a \varepsilon L$. $[a]\theta$ is a convex sublattice. **[5]**

Q3) a) Let L and k be the lattices. Let θ be a congruence relation of L and let ϕ be a congruence relation of k. Define the relation $\theta \times \phi$ on $L \times K$ by **[7]**

$\langle a,b \rangle \equiv \langle c,d \rangle \ (\theta \times \phi)$ if and only if $a \equiv c\ (\theta)$ and $b \equiv d(\phi)$.

Then prove that $\theta \times \phi$ is a congruence relation on $L \times K$. Also, show that every congruence relation of $L \times K$ is of this form.

P.T.O.

b) Prove that the following inequalities hold in any lattice. **[5]**

i) $(x \wedge y) v (x \wedge z) \leq x \wedge (yvz)$

ii) $(x \wedge y) v (x \wedge z) \leq x \wedge (yv(x \wedge z)$

c) Prove that the dual of a distributive lattice is distributive **[4]**

Q4) a) Let L be a pseudocomplemented meet semilattice, and S(L) = {a*/a∈L)

Then prove that **[6]**

i) a ∈ S(L) if and only if a = a**

ii) a,b∈S(L) implies that a ^ b ∈ S(L).

b) Prove that in a distributive lattice L, if the ideals IVJ and I ^ J are principal, then so are I and J. **[6]**

c) Prove that every distributive lattice is modular, but not conversely. Find the smallest modular but nondistributive lattice. **[4]**

Q5) a) Prove that a lattice L is modular if and only if it does not contain a pentagon. **[8]**

b) Let L be a pseudocomplemented meet-semilattice and let a,b ∈ L. Then verify the formulas **[4]**

(a ^ b)* = (a** ^ b)* = (a** ^ b**)*

c) Prove that every complete lattice is bounded. Is the converse true? Justify.**[4]**

Q6) a) Let L be an finite distributive lattice. Then prove that the map $\phi : a \rightarrow r$ (a) is an isomorphism between L and H (J(L)), the set of all hereditary subsets of the set of all nonzero join-irreducible elements of L. **[8]**

b) Prove that every modular lattice satisfies both the upper and the lower covering conditions. **[5]**

c) Give an example of a lattice which is semi-modular but not modular.**[3]**

Q7) a) State and prove fixed point theorem for complete lattices. **[8]**

b) Let L be a lattice of finite length. If L is semimodular, then prove that any two maximal chains of L are of the same length. **[8]**

Q8) a) Prove that a lattice is distributive if and only if it is isomorphic to a ring of sets. **[8]**

b) Let L be a lattice and $a,b \in L$. If amb in L and bMa in the dual of L.Then prove that $[a \wedge b, b] \cong [a, a \vee b]$ **[8]**

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