Total No. of Questions : 8]

SEAT No. :

P1242

[Total No. of Pages : 2

[5121]-41
M.A./M.Sc.
MATHEMATICS
MT - 801 : Field Theory
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions of the following.*
2) *Figures to the right indicate full marks.*

Q1) a) Let p(x) be an irreducible polynomial in F(x) and 'u' be a root of f(x) in an extension E of F. Then prove that F(u), the subfield of E generated by F and u is the set

$\{b_0 + b_1 u + \text{------} + b_m u^m \,/\, b_0 + b_1 x + \text{----} + b_m x^m \in F[x]\}$

Further show that if degree of p(x) in 'n' then $\{1, u, \text{-----}, u^{n-1}\}$ forms a basis of F(u) over F. **[8]**

b) Define normal extension and illustrate it by an example. **[5]**

c) Is $f(x) = x^2 + x + 1 \in \mathbb{Z}_2[x]$ irreducible over $\mathbb{Z}_2$. **[3]**

Q2) a) If K is a splitting field of $f(x) \in F[x]$ over F then show that K is an algebraic extension of F. **[8]**

b) If $p(x) = x^2 - x - 1 \in \mathbb{Z}_3[x]$, then show that there exist an extension K to $\mathbb{Z}_3$ with nine elements having all roots of $p(x)$. **[8]**

Q3) a) Show that doubling the cube and squaring the circle are impossible by using rural and compass. **[8]**

b) Determine the minimal polynomial over Q for the element $\sqrt{-1+\sqrt{2}}$. **[5]**

c) Construct a field with 4-elements. **[3]**

Q4) a) If E is a Galois extension of F and K is any subfield of E containing F then with usual notation prove that $K = E_{G(E/k)}$. **[8]**

b) Let E be an extension of a field F. Define the group of F-automorphisms of E with an example. **[5]**

c) Examine whether the polynomial $x^4 + x + 1 \in Q(x)$ is a separable polynomial. **[3]**

P.T.O.

Q5) a) If K is a field of characteristic $p \neq 0$ and if K is a perfect field then show that $K^P = K$. **[8]**

b) If $[F(\alpha) : F]$ is odd then prove that $F(\alpha)=F(\alpha^2)$. **[4]**

c) Prove that $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{2}+\sqrt{3}$ are algebraic over Q. **[4]**

Q6) a) Determine the splitting field and it's degree over Q for the polynomial $f(x) = x^4 - 2$. **[8]**

b) Let E be the splitting field of $x^n - a \in F[x]$, then prove that G(E/F) is solvable group. **[8]**

Q7) a) Show that the group $G = G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$ of Q-automorphisms of $Q\left(\sqrt[3]{2}\right)$ is trivial. **[8]**

b) State the fundamental theorem of Galois theory. **[4]**

c) Let $F = Q\left(\sqrt{2}\right)$ and $E = Q\left(\sqrt[4]{2}\right)$. Show that E is a normal extension of F and F is a normal extension of Q but E is not a normal extension of Q. **[4]**

Q8) a) Show that a finite field F of p^n elements has exactly one subfield with p^m elements for each divisor m of n. **[6]**

b) Determine Galois group of $f(x) = x^3 - x + 1$ EQ$[x]$ over Q. **[5]**

c) Show that $Q\left(\sqrt{2},\sqrt{3}\right) = Q\left(\sqrt{2}+\sqrt{3}\right)$. **[5]**

⊖ ⊖ ⊖