Total No. of Questions : 8]

SEAT No. :

P1244

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M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT - 803 : Differential Manifolds

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let F be a k-tensor. With usual notation, if $AF = \sum_{\sigma \in s_k} (sign\sigma) F^{\sigma}$, then prove that AF is an alternating tensor. Find AF if F is already alternating. **[7]**

b) Show that g(X,Y,Z) = det $\begin{pmatrix} x_i & y_i & z_i \\ x_j & y_j & z_j \\ x_k & y_k & z_k \end{pmatrix}$ is an alternating 3-tensor on $\mathbb{R}^n$.

Further, express g as a combination of elementary tensors. **[6]**

c) Define : Volume of parametrized surface in $\mathbb{R}^n$. **[3]**

Q2) a) Let U be an open set in $\mathbb{R}^n$ and f : U $\rightarrow$ $\mathbb{R}^n$ be of class C^r. Let $M = < x : f(x) = 0\}$ and $N = < x : f(x) \geq 0\}$. If M is non-empty and Df(x) has rank one at each point of M, then prove that N is an n-manifold in $\mathbb{R}^n$ and ∂N = M. **[8]**

b) Define an exact form and give one example. **[4]**

c) Give an example of a 2-manifold in $\mathbb{R}^3$ without boundary. **[4]**

Q3) a) Define the differential operator d. For any k-form w, show that d(dw) = 0. **[7]**

b) If $w = x^2\, dx + y dy + z e^x\, dz$ and $\eta = y\cos x\, dx + x dy + 2xy\, dz$, then find $(w \wedge \eta)$ **[5]**

c) Define 'Alternating Tensor' and give an example. **[4]**

P.T.O.

Q4) a) Define orientation of a manifold M and induced orientation on ∂M. **[4]**

b) State Green's theorem for compact, oriented 2-manifold. **[4]**

c) Let $\alpha : (0,1)^2 \to \mathbb{R}^3$ be given by $\alpha(u,v) = (u, v, u^2 + v^2 + 1)$ **[8]**

Let Y be the image set of α.

Evaluate $\int_Y x_2 dx_2 \wedge dx_3 + x_1 x_3 dx_1 \wedge dx_3$

Q5) a) Let M be a k-manifold in $\mathbb{R}^n$. If ∂M is non-empty, then prove that ∂M is k-1 manifold without boundary. **[7]**

b) If $w = x^2 z^2\, dx + 2(\cos y)^z\, dy + e^z\, dz$, then find dw. **[5]**

c) Show that a unit n-ball B^n is an n-manifold in $\mathbb{R}^n$

What is its boundary? **[4]**

Q6) a) With usual notation, show that $\alpha^*(dw) = d(\alpha^* w)$ **[8]**

b) Let $A = \mathbb{R}^2 - \{0\}$. If $w = \frac{xdx + ydy}{x^2 + y^2}$, **[8]**

then show that w is closed and exact on A. **[8]**

Q7) a) If $T : V \to W$ is a linear transformation, and f, g are alternating tensors on W; then prove that $T^* (f \wedge g) = T^* F \wedge T^* g$. **[8]**

b) Let $w = y^2 z\, dx + x^2 z dy + x^2 y dz$ and $\alpha(u,v) = (u-v, uv, u^2)$ Find $\alpha^*(dw)$. **[8]**

Q8) a) State Stokes' theorem **[4]**

b) Define closed form and give an example **[4]**

c) What is the dimension of the space of alternating k-tensors on an n-dimensional vector space V, denoted as $A^k(V)$? Justify your answer. **[8]**

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