Total No. of Questions : 8]

SEAT No. :

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[Total No. of Pages : 3

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M.A/M.Sc.

MATHEMATICS

MT - 803 : Differential Manifolds

(2008 Pattern) (Semester-IV)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define volume of a parametrized manifold and prove that it is invariant under reparametrization. **[6]**

b) Give an example of an alternating tensor on $\mathbb{R}^n$. **[5]**

c) State the generalized Stokes theorem. **[5]**

Q2) a) Define the differential operator d and prove that for f: $\mathbb{R}^n \to \mathbb{R}$ of class C^∞, $df = \frac{\partial f}{\partial x_1} dx_1 + \ldots + \frac{\partial f}{\partial x_n} dx_n$ **[6]**

b) Let f, g be tensors on $\mathbb{R}^4$ defined by

$f(x, y, z) = x_1 y_2 z_3 - x_2 y_3 z_1$ and

$g(u,v) = 2\ u,v_3 - u_4\ v_2$

Find $f \otimes g$ and express it as a linear combination of elementary 5-tensors **[5]**

c) If $\eta = x\, y\, dx + e^y\, x\, z^2\, dy + x \cos y\, dz$, then find d η. **[5]**

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Q3) a) Let $k > 1$. If M is an orientable k-manifold with non-empty boundary, then prove that ∂M is orientable. **[7]**

b) If $w = yz\,dx + x\,z\,dy + xy\,dz$ and $\eta = e^y\, x\, dx + \sin x \,.\, z\, dy + xyz\, dz$, then find $w \wedge \eta$ **[5]**

c) Find the centroid of the parametrised curve α (t) = (a cost, a sint), $0 < t < \pi$ **[4]**

Q4) a) Define 'Exact form' and give an example. **[6]**

b) State Green's theorem for 2-manifolds in $\mathbb{R}^2$ **[5]**

c) Let $w = x\,y\,dx + z^2\,dy + yz\,dz$ and α (u,v) = (u+v, u–v, uv). Find α^*(dw) **[5]**

Q5) a) For any k- form w, prove that d(dw)=0 **[8]**

b) Let $A = (0, 1)^3$. Let α: $A \rightarrow \mathbb{R}^4$ be given by the equation α (s, t, u)= (s, u, t,$(2u–t)^2$).Let Y be the image set of α. Evaluate the integral over Y_α of the 3-form

$$x_1\, dx_1 \wedge dx_4 \wedge dx_3 + 2x_2x_3\, dx_1 \wedge dx_2 \wedge dx_3.$$ **[8]**

Q6) a) Let A be an set in $\mathbb{R}^k$. Let α: $A \rightarrow \mathbb{R}^n$ be of class C^∞. If w is an l-form defined in an open set of $\mathbb{R}^n$ containing α (A), the prove that α^*(dw) $= d(\alpha^* w)$ **[8]**

b) Give an example of a 3-manifold in $\mathbb{R}^3$. **[4]**

c) Find area of the 2-sphere $s^2(a)$ of radius a in $\mathbb{R}^3$ **[4]**

Q7) a) With usual notation, prove that

$d\,(w \wedge \eta) = dw \wedge \eta + (-1)^k\, w \wedge d\,\eta$ where w and η are forms of order k and *l* respectively **[8]**

b) Justify whether true or false: If *f* and *g* are alternating tensors, then $f \otimes g$ is also an alternating tensor. **[4]**

c) Define 'closed form' and give an example. **[4]**

Q8) a) Define boundary of a manifold and give an example of a 2-manifold in $\mathbb{R}^3$ without boundary. **[6]**

b) For a k-tensor F on $\mathbb{R}^n$, if

$AF = \sum_{6} (\text{sign}\, 6)\, F^6$, then prove that AF is an alternating tensor **[5]**

c) Let $\alpha : \mathbb{R}^3 \to \mathbb{R}^6$ be given by $\alpha\,(x, y, z) = (x^2y, y^2z, x^2z, x, y, z)$

Find $d\alpha_1 \wedge d\alpha_3 \wedge d\alpha_5$ **[5]**

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