Total No. of Questions : 8]

SEAT No. :

P1403

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[5221]-42
M.A./M.Sc.
MATHEMATICS
MT-802 : Combinatorics
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) How many arrangements of the letters in 'MISSISSIPPI' in which **[6]**

i) The M is immediately followed by an I?

ii) The M is beside an I, that is, an I is just before or just after the M.

b) Prove by combinatorial argument that $C(r,r) + C(r+1,r) + C(r+2,r)+$ .....$+ C(n,r) = C(n+1,r+1)$. Hence evaluate the sum $1^2 + 2^2 +$ .....$+ n^2$. **[6]**

c) Find two different chessboard(not row or column rearrangements of one another) that have the same rook polynomial. Also, write the rook polynomial. **[4]**

Q2) a) What fraction of all arrangements of 'INSTRUCTOR' have three consecutive vowels? **[6]**

b) How many arrangements are there of 'TAMELY' with either T before A, or A before M or M before E? [By before, we mean anywhere before, not just immediately before). **[6]**

c) How many nonnegative integer solutions are there to the pair of equations $x_1 + x_2 +$.....$+ x_6 = 20$ and $x_1 + x_2 + x_3 = 7$? **[4]**

P.T.O.

Q3) a) How many ways are there to place an order for 12 chocolate sundaes if there are 5 types of sundaes and at most 4 sundaes of one type are allowed? **[6]**

b) Find ordinary generating function whose coefficient a_r equals $3r^2$. Hence, evaluate the sum $0 + 3 + 12 + \ldots\ldots\ldots\ldots + 3n^2$. **[6]**

c) How many distributions of 24 different objects into three different boxes are there with twice as many objects in one box as in the other two combined? **[4]**

Q4) a) Using generating function, find the number of distributions of 18 chocolate bunny rabbits into four Easter baskets with atleast 3 rabbits in each basket. **[6]**

b) How many ways are there to collect \$24 from 4 children and 6 adults if each person gives atleast \$1, but each child can give at most \$4 and each adult at most \$7? **[6]**

c) Show that in any set of n integers, $n \geq 3$, there always exists a pair of integers whose difference is divisible by n–1. **[4]**

Q5) a) How many n-digit decimal sequences (using digits 0,1,2.....9) are there in which digits 1,2,3 all appear? **[6]**

b) How many ways are there to make an r-arrangements of pennies, nickels, dimes and quarters with at least one penny and an odd number of quarters? **[6]**

c) Solve the recurrence relation $a_n = 2a_{n/2} + 2, n \geq 4$, with $a_2 = 1$. **[4]**

Q6) a) Uisng generating functions, solve the recurrence relation,

$a_n = 2a_{n-1} + 2^n$ with $a_0 = 1$ **[6]**

b) How many ways are there to assign 20 different people to three different rooms with atleast one person in each rooms? **[6]**

c) Solve the recurrence relation **[4]**

$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, with $a_0 = a_1 = 1; a_2 = 2$

Q7) a) Find recurrence relation for the number of n-digit binary sequences with an even number of zeros. **[6]**

b) Suppose a school with 120 students offers Yoga and Karate. If the number of students taking Yoga alone is twice the number taking Karate(Possibly, Karate and Yoga), if 25 more students study neither skill than study both skills, and if 75 students take at least one skill, then how many students study Yoga? **[6]**

c) How many arrangements of the letters in 'MATHEMATICS' are there in which TH appear together but the TH is not immediately followed by an E? **[4]**

Q8) a) Five officials $O_1, O_2, \ldots\ldots\ldots, O_5$ are to be assigned five different city cars $C_1, C_2, \ldots\ldots\ldots\ldots, C_5$. If O_1 will not drive C_1 or C_5; O_2 will not drive C_2 or C_3; O_3 will not drive C_3; O_4 will not drive C_1 or C_5; O_5 will not drive C_3. How many ways are there to assign the officials to different cars? **[8]**

b) Solve the recurrence relation $a_n^2 = 2a_{n-1}^2 + 1$ with $a_0 = 1$. **[4]**

c) Show that if n+1 distinct numbers are chosen from 1,2,.....2n, then two of the numbers must always be consecutive integers. **[4]**

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