Total No. of Questions : 8]

SEAT No. :

P1241

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M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-705 : Graph Theory

(2008 Pattern)

Time : 3 Hour] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that every set of six people contains at least three mutual acquaintances or three mutual strangers. **[6]**

b) Prove that every graph with n vertices and k edges has at least $n-k$ components. **[4]**

c) Prove that an edge is a cut edge if and only if it belongs to no cycle. **[6]**

Q2) a) Find the number of simple graphs with a vertex set X of size n. **[5]**

Draw all the non isomorphic simple graphs on a fixed set of four vertices.

b) Prove that the Petersen graph has ten 6-cycles. **[5]**

c) Prove that a graph is bipartite if it has no odd cycle. **[6]**

Q3) a) Prove that for a connected nontrivial graph with exactly 2k odd vertices, the minimum number of trails that decompose it is $max\{k, 1\}$. **[8]**

b) Show that if G is a simple n- vertex graph with $\delta(G) \geq \frac{(n-1)}{2}$, then G is connected **[4]**

c) Show that the number of vertices in a self-complementary graph is either $4k$ or $4k + 1$, where k is a positive integer. **[4]**

P.T.O.

Q4) a) Prove that if T is a tree with k edges and G is a simple graph with $\delta(G) \geq k$ then T is a subgraph of G. **[7]**

b) Show that every graph has an even number of vertices of odd degree. **[3]**

c) Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph. **[6]**

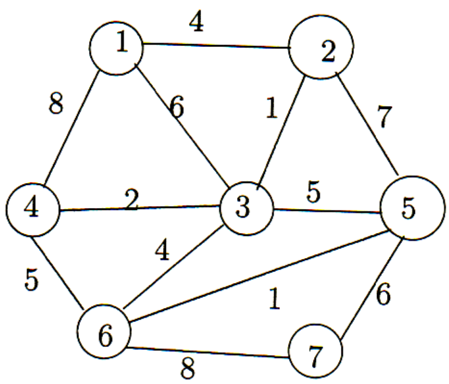
Q5) a) Prove that the center of a tree is a vertex or an edge. **[7]**

b) Let T be a tree with average degree a. Determine $n(T)$ in terms of a. **[3]**

c) Prove that if G is a simple graph with *diam* $G \geq 3$, then diam $\bar{G} \leq 3$. **[6]**

Q6) a) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **[10]**

b) Use Dijkstra's algorithm to find shortest distance from a vertex 1 to any other vertex in the following graph **[6]**



Q7) a) Prove that in a connected weighted graph G, Kruskal's Algorithm constructs a minimum-weight spanning tree. **[6]**

b) Determine whether the sequence (5, 5, 4, 3, 2, 2, 2, 1) is graphic. Provide a construction or a proof of impossibility. **[4]**

c) Show that the connectivity of the hypercube Q_k is k. **[6]**

Q8) a) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. **[5]**

b) Define clique number and independence number of a graph G with an example. Prove that for every graph G, $\chi(G) \geq \omega(G)$ and

where $\omega(G)$ is the clique number of G and $\propto(G)$ is the independence number of G. **[5]**

c) Prove that if G is a connected graph, then an edge cut F is a bond if and only if G - F has exactly two components. **[6]**

$$\chi(G) \geq \frac{n(G)}{\alpha(G)},$$