Total No. of Questions :8]

SEAT No. :

P1395

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M.A./M.Sc.

MATHEMATICS

MT- 605: Partial Differential Equations

(2008 Pattern) (Old) (Semester - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the parameters 'a' and 'b' from the equation $2Z = (ax + y)^2 + b$. **[5]**

b) Explain the method of solving the following first order partial differential equations. **[6]**

i) $f(z, p, q) = 0$

ii) $g(x, p) = h(y, q)$

c) Find the general integral of: $z(xp - yq) = y^2 - x^2$. **[5]**

Q2) a) If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$, then prove that h_1 and h_2 satisfy: **[5]**

$$\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_z)} = 0 \text{ where } h = h_i,\ i = 1, 2.$$

b) Verify that the equation is integrable and find its solution

$$yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$$ **[6]**

c) Find the complete integral of the partial differential equation: **[5]**

$$(1+yz)dx + z(z-x)dy - (1+xy)dz = 0$$

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Q3) a) Show that the equations: $f = xp - yq - x = 0,\ g = x^2 p + q - xz = 0$ are compatible and find a one parameter family of common solution. **[6]**

b) Find the complete integral of the equation: $p^2 x + q^2 y = z$, by Jacobi's method. **[5]**

c) Find the complete integral of the equation: $px + qy = pq$ by Charpit's method. **[5]**

Q4) a) Find the solution of the equation: **[8]**

$$Z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y) \text{ which passes through x-axis.}$$

b) Reduce the equation: **[5]**

$$y^2 u_{xx} - 2\,xy\,u_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y \text{ to canonical form and solve it.}$$

c) Derive the analytic expression for Monge cone at (x_0, y_0, z_0). **[3]**

Q5) a) Find the integral surface of the equation: $pq = z$, passing through $\mathrm{C}: x_0 = 0, y_0 = s, \mathrm{z}_0 = s^2$. **[5]**

b) Solve by Jacobi's method: $z^2 + 2\mathrm{U}_z - \mathrm{U}_x^2 - \mathrm{U}_y^2 = 0$. **[6]**

c) State and prove Harnack's theorem. **[5]**

Q6) a) Prove that the solution of Neumann problem is unique upto addition of a constant. **[8]**

b) Find the solution of Heat-equation in finite rod, which is defined as: **[8]**

$$u_t = x\,u_{xx}, 0 < x < e, t > 0$$
$$u(0,t) = u(e,t) = 0, t > 0$$
$$u(x,0) = f(x),\ 0 \le x \le e.$$

Q7) a) State Dirichilet's problem for a circle and find it's solution. **[8]**

b) State and prove Kelvin's Inversion theorem. **[8]**

Q8) a) Using Duhamel's principle find the solution non-homogenous wave-equation: **[8]**

$$u_{tt} = x^2 u_{xx}, F(x,t), -\infty < x < \infty, t > 0$$
$$u(x,0) = u_t(x,0) = 0, -\infty < x < \infty$$

b) Classify the following equation in to Hyperbolic, Parabolic or Elliptic type. **[8]**

i) $7u_{xx} - 10u_{xy} - 22u_{yz} + 7u_{yy} - 16u_{zz} - 5u_{zz} = 0$

ii) $e^z u_{xy} - u_{xx} = \log(x^2 + y^2 + z^2 + 1)$

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