Total No. of Questions : 8]

SEAT No. :

P1256

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M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-605 : Partial Differential Equations

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the arbitrary function 'F' from the equation **[4]**

$$x + y + z = F(x^2 + y^2 + z^2)$$

b) Find the general integral of equation. **[4]**

$$z(xp - yq) = y^2 - x^2$$

c) Define the following terms and example of each **[2]**

i) Linear equation.

ii) Quasi-linear equation

Q2) a) Verify that the equation is integrable and find it's solution. **[4]**

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

b) Show that the equations : $p^2 + q^2 = 1$ and $(p^2 + q^2)x = pz$. **[4]**

are compatible and solve them.

c) Find the complete integral of : the partial differential equation. **[2]**

$$pqz = p^2(xq + p^2) + q^2(yp + q^2)$$

P.T.O.

Q3) a) If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$, then prove that h_1 and h_2 satisfy : **[4]**

$$\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_z)} = 0$$

b) Find the complete integral of : **[4]**

$$(1+yz)dx + x(z-x)dy - (1+xy)dz = 0$$

c) Solve the equation : **[2]**

$$(x^2 z - y^3)dx + 3xy^2 dy + x^3 dz = 0$$

Q4) a) Find the complete integral of first order partial differential equation : $z^2(p^2z^2 + q^2) = 1$ by Charpit's method. **[4]**

b) Find the general integral of the partial differential equation : $(x-y)y^2 p + (y-x)x^2 q = (x^2 + y^2)$ and particular solution through, $xz = a^2, y = 0$. **[4]**

c) Derive the analytic expression for the monge cone at (x_0, y_0, z_0), **[2]**

Q5) a) Find the integral surface of the equation, $pq = z$; passing through curve C : $x_0 = 0,\ y_0 = s, z_0 = s^2$ **[4]**

b) Reduce the equation to canonical form the solve it $u_{xx} + 2u_{xy} + 17u_{yy} = 0$. **[4]**

c) Find the initial strip for the equation : $pq = xy$ which passes through the curve C : $z = x$, $y = 0$. **[2]**

Q6) a) If $u(x,y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$. Then u attains it's maximum on the boundary B of D. **[4]**

b) State and prove Harnack's theorem. **[4]**

c) Classify the following equation into hyperbolic, parabolic or elliptic type : $e^z u_{xy} - u_{xx} = \log(x^2 + y^2 + z^2 + 1)$. **[2]**

Q7) a) Using D-Alembert's solution of infinite string find the solution of:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, 0 < x < \infty, t > 0$$

$$y(x,0) = u(x), yt(x,0) = V(x), x \geq, 0$$

$$y(0,t) = 0,\ t \geq, 0$$ **[5]**

b) State and prove Kelvin's Inversion Theorem. **[5]**

Q8) a) Using Duhamel's principle find the solution of non-homogenous Heat equation : $u_t + ku_{xx} = f(x,t), -\infty < x < \infty, t > 0$.

$$u(x,0) = 0, \quad -\infty < x < \infty.$$ **[5]**

b) Find the solution of : $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta Q} = 0,\ r < a$ subject to the boundary

$$\frac{\partial u}{\partial r} = f(\theta) \text{ on } r = a, \int_0^{2\pi} f(\theta) d\theta = 0.$$ **[5]**

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