Total No. of Questions :8]

SEAT No. :

P1910

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[4921]-2005
M.A/M.Sc.
MATHEMATICS
MT-605: Partial Differential Equations
(2013 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the arbitrary function F from the equation $F(x-z, y-z)=0$ and find the corresponding partial differential equation. **[3]**

b) Define the following terms and example of each **[2]**

i) Linear equation.

ii) Quasi - linear equation.

c) Find the general integral of **[5]**

$$y^2 p - xyq = x(z-2y).$$

Q2) a) Prove that the Pfaffian differential equation **[5]**

$\bar{X}\cdot d\bar{r} = P(x,y,z)dz + Q(x,y,z)dy + R(x,y,z)dz = 0$ is integrable if and only if $\bar{X}\cdot \text{curl } \bar{X} = 0$.

b) Find the general integral of the partial differential **[3]**

$(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x = 1, y = 0$.

c) Show that the equations $xp - yq = x$ and $x^2 p + q = xz$ are compatible. **[2]**

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Q3) a) Explain the method of solving the following first order partial differential equation $g(x, p) = h(y, q)$. **[4]**

b) Find a complete integral of $f = x^2 - pqxy = 0$ by Charpit's method. **[4]**

c) Find a complete integral of the partial differential equation **[2]**

$$pqz = p^2(xq + p^2) + q^2(yp + q^2).$$

Q4) a) Explain the Jacobi's method for solving a partial differential equation **[5]**

$$f(x, y, z, u_x, u_y, u_z) = 0.$$

b) Find a one parameter family of common solutions of the equtions $xp = yq$ and $z(xp + yq) = 2xy$. **[3]**

c) Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$. **[2]**

Q5) a) Solve the Cauchy problem for $2z_x + yz_y = z$ for the initial data curve $C : x_0 = s, y_0 = s^2, z_0 = 1,\ 1 \le s \le 2$. **[3]**

b) Reduce the equation $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$ to canonical form. **[3]**

c) Prove that the solution of the Dirichlet problem, if it exists is unique. **[4]**

Q6) a) Using D' Alemberts solution of infinite string find the solution of **[5]**

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}, 0 < x < \infty, t > 0$$

$$y(x,0) = u(x),\ \ y_t(x,0) = v(x),\ \ x \ge 0$$
$$y(0,t) = 0,\ \ t \ge 0$$

b) State and prove Harnack's theorem. **[3]**

c) Is the surface $x^2 + y^2 + z^2 = cx^{2/3}$ equipotential? If yes, then find potential function. **[2]**

Q7) a) Using the variable separable method solve $u_t = ku_{xx}; 0 < x < a,\ t > 0$ which satisfies condition $u(0,t) = u(a,t) = 0;\ t > 0$ and $u(x,0) = x(a-x);\ 0 \le x \le a$. **[5]**

b) To find solution of $\nabla^2 u = u_{xx} + u_{yy} = 0, -\infty < x < \infty, y > 0$

$u_y(x,0) = g(x), -\infty < x < \infty$ with the conditions that $u\ (x, y)$ is bounded as $y \to \infty$, u and u_x vanish as $|x| \to \infty$ and $\int_{-\infty}^{\infty} g(x)dx = 0$. **[5]**

Q8) a) State and prove Kelvin's inversion theorem. **[5]**

b) To find solution of $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0\ \ r < a$, subject to the boundary **[5]**

$\frac{\partial u}{\partial r} = f(\theta)$ on $r = a$, $\int_0^{2\pi} f(\theta)d\theta = 0$.

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