Total No. of Questions :8]

SEAT No. :

P1889

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[4921] - 203
M.A. / M.Sc.
MATHEMATICS
(MT - 603) : Groups and Rings
(2008 Pattern) (Semester -II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If G be a finite group and $a, b \in G$, then prove the order (ab) = order (ba). **[5]**

b) Define a subgroup. Let H be finite subset of a group G. Then prove that H is a subgroup of G if and only if H is closed under the operation of G. **[5]**

c) Prove that any cyclic group is isomorphic to either $\mathbb{Z}_n$ for some $n \in \mathbb{N}$ or $\mathbb{Z}$. Also find all the generators of $\mathbb{Z}_{25}$. **[6]**

Q2) a) Prove that $(\mathbb{Z}[x],+)$ and $(\mathbb{Q}^+,.)$ are isomorphic. **[5]**

b) Give an example of an infinite group whose every element is of finite order. Justify the answer. **[5]**

c) Prove that every subgroup of a cyclic group is cyclic. Moreover prove that, if $|< a >| = n$, then the order of any subgroup of $< a >$ is a divisor of n; and, for each positive divisor k of n, the group $< a >$ has exactly one subgroup of order k. **[6]**

Q3) a) Define Centralizer of an element of a group G. prove that for each a in group G, the centralizer of a is a subgroup of G. **[5]**

b) Prove that is G if a group, then set of automorphisms of G, A*ut* (G) is a group. **[5]**

c) Find A*ut* $(\mathbb{Z}_4)$, the group of automorphisms of $\mathbb{Z}_4$. **[6]**

Q4) a) Find the inverse and the order of each of the following permutations in S_{12} **[5]**

i) (5 3 4 7) (2 1 6)

ii) (9 2 10 4) (7 1 5) (3 8 12).

b) Prove that set of even permutations A_n forms a subgroup of S_n. Also prove that for $n > 1$, A_n has order $n!/2$. **[5]**

c) Let G be a finite group and p be a prime. If p^k divides $|G|$, then prove that G has atleast one subgroup of order p^k. **[6]**

Q5) a) State and prove the first isomorphism theorem. **[5]**

b) Give an example of a non abelian group whose all proper subgroups are abelian. **[5]**

c) If $\tau = (1\ 5\ 4)\ (2\ 7)$, $\rho = (1\,2\,8\,7\,5\,6)(3\,4) \in S_8$. Then find $\tau^{-1}\rho\tau$ and $\rho^{-1}\tau\rho$. **[6]**

Q6) a) Prove that every group is isomorphic to a subgroup of S_n for some $n \in \mathbb{N}$. **[5]**

b) Determine all the homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$. **[5]**

c) Find all the non isomorphic abelian groups of order 2016. **[6]**

Q7) a) Determine all the groups of order 15. **[5]**

b) Let H be an index 2 subgroup of group G. Prove that $a^2 \in H, \forall a \in G$. **[5]**

c) Let G be a finite group of permutations of *a* set S. Then prove that for any *i* from S, $|orb_G(i)| = |stab_G(i)|$. **[6]**

Q8) a) If G is a finite abelian group, then what is the product of all the elements in G? Justify your answer. **[5]**

b) Define homomorphism and kernel of a homomorphism. Prove that every normal subgroup of a group G is the kernel of a homomorphism of G. **[5]**

c) Prove that the groups of order 20 and 27 are not simple. **[6]**