Total No. of Questions :8]

SEAT No. :

P1391

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[5221]-21
M.A./M.Sc.
MATHEMATICS
MT - 601 : General Topology
(2008 Pattern) (Old course) (Semester - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $\mathbb{B}$ be a collection of non-empty sets (not necessarily disjoint). Prove that the exists a function

$$c : \mathbb{B} \to \bigcup_{B\in\mathbb{B}} B$$

Such that c(B) is an element of B, for each $B \in B$. **[8]**

b) Prove that the topology $\mathbb{R}_l$ on $\mathbb{R}$ is strictly finer than the standard topology on $\mathbb{R}$. **[4]**

c) Suppose X and Y are two topological spaces. Show that the collection.

$$S = \{\pi_1^{-1}(\text{U}) | \text{U is open in X}\} \cup \{\pi_2^{-1}(\text{V}) | \text{V is open in Y}\}$$

forms a sub basis for product topology on X×Y. **[4]**

Q2) a) If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\bigcap \tau_\alpha$ is a topology on X. Is $\bigcup \tau_\alpha$ a topology on X. Justify. **[6]**

b) Prove that the order topology on $\mathbb{Z}_+$ is the discrete topology on $\mathbb{Z}_+$. **[6]**

c) Let Y = [–1,1],which of the following sets are open in Y? Which are open in $\mathbb{R}$? Justify.

P.T.O.

i) $A=\left\{x \mid \frac{1}{2} \leq |x| \leq 1\right\}$

ii) $B=\left\{x \mid \frac{1}{2} < |x| \leq 1\right\}$. **[4]**

Q3) a) Let A be a subset of the topological space X. Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A. **[6]**

b) Let X be a space satisfying T_1 axiom and A be a subset of X. Prove that the point x is a limit point of A if and only if every neighbourhood of x contains lnfinitely many points of A. **[6]**

c) Show that every simply ordered set is a Hausdorff space in the order topology. **[4]**

Q4) a) Prove that every finite point set in a Hausdorff space X is closed. **[6]**

b) Show that the topological space X is Hausdorff if and only if the diagonal $\Delta=\{x \times x \mid x \in X\}$ is closed in $X \times X$. **[6]**

c) State and prove the pasting lemma. **[4]**

Q5) a) Let A be any set, X and Y be two topological spaces with $f: A \rightarrow X \times Y$ defined by $f(a)=(f_1(a), f_2(a))$. Then show that f is continuous if and only if f_1 and f_2 are continuous, where $f_1: A \rightarrow X$ and $f_2: A \rightarrow Y$. **[8]**

b) Prove that the image of a connected space under a continuous map is connected. **[4]**

c) Let $\{A_n\}$ be a sequence of connected subspaces of X such that $A_n \cap A_{n+1} \neq \phi$, for all n. Show that $\bigcup_n A_n$ is connected. **[4]**

Q6) a) Prove that a topological space X is locally connected if and only if for every open set $\cup$ of X, each component of $\cup$ is open in X. **[6]**

b) Show that the compact subspace of a Hausdorff is closed. **[6]**

c) Define quotient topology and give an example of a quotient map which is not an open map. **[4]**

Q7) a) State and prove the tube lemma. **[8]**

b) Prove that compactness implies limit point compactness. Is converse true? Justify. **[5]**

c) Show that not every first countable space is second countable. **[3]**

Q8) a) State and prove the Tychonoff theorem. **[12]**

b) State : **[4]**

i) Tietze extension theorem.

ii) The Urysohn lemma.