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SEAT No. :

P1412

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M.A./M.Sc.

MATHEMATICS

MT - 601:Complex Analysis

(2013 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt ANY FIVE questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $f(z)=f(x+iy)=\sqrt{|x||y|}, x, y \in \mathbb{R}$ then show that the function f satisfy C.R. equations at origin but f is not holomorphic at origin. **[5]**

b) If f is holomorphic in a region Ω and $f'=0$ then prove that f is constant function. **[3]**

c) Find radius of convergence of the series $\sum_{n=0}^{\infty}\frac{n^2}{4^n+3n}z^n$. **[2]**

Q2) a) Show that the power series $\sum_{n=1}^{\infty} nz^n$ does not converge on any point of the unit circle. **[5]**

b) If γ be a smooth curve in $\mathbb{C}$ parametrized by $z(t)=[a,b]\to\mathbb{C}$ and γ^- denote the curve with same image as γ but with opposite orientation then prove that, $\int_{\gamma} f(z)dz = -\int_{\gamma^-} f(z)dz$. **[3]**

c) If f is continuous function in region Ω then prove that any two primitive of f differe by a constant. **[2]**

Q3) a) If f is holomorphic in an open set Ω that contains a rectangle R and it's interior then prove that $\int_{R} f(z)dz = 0$. **[5]**

b) If f is holomorphic function in Ω^+ that extend continuously to I and such that f is real valued on I then prove that there exists a function F holomorphic in all of Ω such that F = f on Ω^+. **[3]**

c) State symmetric principle. **[2]**

P.T.O.

Q4) a) If f is holomorphic in an open set Ω. If D is a disc centered at z_0 and whose closure is contained in Ω then prove that f has a power series expansion at z_0, $f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n \; \forall Z \in D$ where

$$a_n = \frac{f^{(n)}(Z_0)}{n!}, \forall n \geq 0.$$ **[5]**

b) Show that every non-constant polynomial $P(z)=a_n z^n + --- + a_n$ with complex coefficient has a root in $\mathbb{C}$. **[3]**

c) State Runge's approximation theorem. **[2]**

Q5) a) If $\{fn\}_{n=1}^{\infty}$ is a sequence of holomorphic function that converges uniformly to a function f in every compact subset of Ω then prove that f is holomorphic in Ω. **[3]**

b) Show that $\int_0^{\infty} \frac{1-\cos x}{x^2} dx = \frac{\pi}{2}$ **[5]**

c) Find the nature of isolated singularity of origin for the function $f(z)=\frac{\sin z}{z}$. **[2]**

Q6) a) If f has a pole of order n at z_0, then prove that

$$f(z)=\frac{a_{-n}}{(z-z_0)^n}+\frac{a_{-n+1}}{(z-z_0)^{n-1}}+---+\frac{a_{-1}}{(z-z_0)}+G(z)$$

Where G is a holomorphic function in a neighborhood of Z_0. **[5]**

b) If f and g are holomorphic in an open set containing a circle C and it's interior and $|f(z)| > |g(z)|$ for all Z in C then prove that f and $f+g$ have the same number of zeros inside a circle C. **[3]**

c) State morera's theorem. **[2]**

Q7) a) Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$. **[5]**

b) If f is holomorphic in an open set containing a circle C and it's interior except for poles at the points Z_1, Z_2, ---- Z_N inside C then prove that

$\int_C f(z)dz = 2\pi i \sum_{k=1}^{N} res_{\Sigma} f$. **[5]**

Q8) a) Show that the complex zeros of sin πz are exactly at the integers and each of order one.

Also find residue of $\frac{1}{\sin \pi z}$ at $z = n \in \mathbb{Z}$. **[5]**

b) Let $D = \{z \in \mathbb{C} / |z| = 1\}$ and $f : D \to D$ be holomorphic function then prove that $|f'(z)| \leq \frac{1}{1-|z|}$ $\forall Z \in D$. **[5]**

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