Total No. of Questions : 4] **SEAT No. :**

P851 **[Total No. of Pages : 8**

[5315]-451
T.Y. B.Sc. (Semester - IV)
STATISTICS (Principal) (Paper - V)
ST - 345 (A) : Reliability and Survival Analysis
(2013 Pattern)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Use of scientific calculator and statistical tables is allowed.***
4) ***Symbols and abbreviations have their usual meaning.***

Q1) Attempt each of the following:

a) Choose the correct alternative in each of the following : **[1 each]**

i) For a k-out-of-n : G system, the number of cut vectors is

A) $2^n - \sum_{r=k}^{n} {}^nC_r$

B) $2^n - \sum_{r=0}^{k-1} {}^nC_r$

C) $\sum_{r=k}^{n} {}^nC_r$

D) $\sum_{r=0}^{k-1} {}^nC_r$

ii) The number of irrelevant components in a series system is

A) n B) 0

C) 1 D) n –1

P.T.O.

iii) Let r(t) be a failure rate function of a lifetime random variable (r.v.) then

A) $r(t)=\lim_{0<h\to\infty}\left(\frac{\bar{F}(t)-\bar{F}(t+h)}{h\,\bar{F}\,(t)}\right)$

B) $r(t)=\lim_{0<h\to 0}\left(\frac{\bar{F}(t+h)-\bar{F}(t)}{h\,\bar{F}\,(t)}\right)$

C) $r(t)=\lim_{0<h\to 0}\left(\frac{\bar{F}(t)-\bar{F}(t+h)}{h\,\bar{F}\,(t)}\right)$

D) $r(t)=\lim_{0<h\to\infty}\left(\frac{\bar{F}(t+h)-\bar{F}(t)}{h\,\bar{F}\,(t)}\right)$

iv) If T is a lifetime r.v. having exponential distribution with mean 5 then mean residual life of unit aged t is

A) 5 B) $\frac{1}{5}$

C) $\frac{1}{5+t}$ D) 5 + t

b) State whether each of the following statement is true or false : **[1 each]**

i) Let h($\underline{P}$) be the system reliability of a coherent system then $0 < h(\underline{P}) < 1$.

ii) Exponential distribution is a member of positive ageing class of lifetime distributions.

c) Define the following terms: **[1 each]**

i) Harmonically New Better than used in Expectation class of lifetime distribution.

ii) Decreasing Failure Rate in Average (DFRA) class of lifetime distribution.

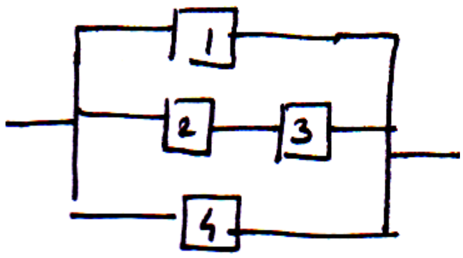
d) Attempt each of the following : **[1 each]**

i) Give an application of a k - out - of - n : G system.

ii) Give an example of right random censoring scheme.

Q2) Attempt any two of the following: **[5 each]**

a) Consider the system with reliability block diagram as given below :



i) Find minimal path sets

ii) Find minimal cut sets

iii) Represent the above system as a series arrangement of minimal cut parallel structure.

iv) Draw reliability block diagram of a dual of a given system.

b) Let h(.) be the reliability function of a coherent system then show that

$h(\underline{P} \coprod \underline{P}') \geq h(\underline{P}) \coprod h(\underline{P}')$ for all $0 \leq Pi \leq 1,\ 0 \leq Pi' \leq 1$ and equality holds for all Pi, Pi' iff the system is parallel.

c) Show that if F belongs to Increasing Failure Rate (IFR) class of lifetime distributions then it belongs to Increasing Failure Rate in Average (IFRA) class of lifetime distributions.

Q3) Attempt any two of the following: **[5 each]**

a) Discuss in detail structural importance of a component in a coherent system with the help of an illustration. Also state its use.

b) Let a lifetime r.v.T with cumulative hazard rate function as

$$R(t) = \begin{cases} kt & ; \quad 0 \le t \le 1 \\ \dfrac{t^2}{2} & ; \quad t > 1 \end{cases}$$

Where k is a positive constant

Find

i) the value of k such that T is strictly continuous.

ii) Survival function of T.

iii) Probability density function of T.

c) Obtain an expression for an actuarial estimator of survival function of a lifetime r.v.T.

Q4) Attempt <u>any one</u> of the following:

a) i) Show that k-out-of-n : G system is a coherent system hence show that series system is also a coherent system. **[6]**

ii) Let a lifetime r.v.T follows Weilbull (λ:scale. parameter, γ:shape parameter)

Compute failure rate function. Discuss its different nature for various values of shape parameter γ. **[4]**

b) i) Define Decreasing Mean Residual Life property (DMRL) of a lifetime r.v.T and New Better than used in Expectation (NBUE) class of lifetime distributions. **[6]**

Show that

1) If F $\in$ IFR $\Rightarrow$ F $\in$ DMRL

2) If F $\in$ DMRL $\Rightarrow$ F $\in$ NBUE

ii) What is the component reliability of each of n identical components in a series system if the system reliability is 0.9? **[2]**

iii) Consider the following data of remission time (in weeks) of leukemia patients receiving control treatment.

1, 1, 2, 2, 5, 5, 5, 5, 6, 6, 8, 8, 8, 8, 9, 9, 10, 10

Calculate an estimate of an unbiased estimator of $\overline{F}(9)$ where $\overline{F}(.)$ is a survival function. **[2]**

Total No. of Questions : 4]

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[5315]-451
T.Y. B.Sc. (Semester - IV)
STATISTICS (Principal) (Paper - V)
ST - 345 (B) : Introduction to Stochastic Processes
(2013 Pattern)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator and statistical tables is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the following:

a) Choose the correct alternative in each of the following: **[1 each]**

i) In a Markov chain if all states communicate with each other then it is called

A) irreducible B) reducible

C) discrete D) finite

ii) Let $\{N(t), t \geq 0\}$ be a Poisson process with parameter λ. Mean number of occurrences in an interval of length t is

A) $\frac{1}{\lambda}$ B) λt

C) $\lambda^2 t$ D) $\frac{1}{\lambda t}$

iii) A persistent state is null persistent if the mean recurrence time is

A) finite B) infinite

C) 0 D) 1

iv) A Markov chain is aperiodic, if its every state is

A) periodic B) irreducible

C) persistent D) aperiodic

b) State whether each of the following statements is true or false : **[1 each]**

i) Absorbing state of a Markov chain is always ergodic.

ii) The difference between two independent Poisson processes is also a Poisson process.

c) Define : **[1 each]**

i) Stochastic matrix

ii) Markov chain (M.C.)

d) i) Define stationary distribution of a Markov Chain. **[1]**

ii) Explain One step transition probability matrix of a Markov Chain. **[1]**

Q2) Attempt <u>any two</u> of the following: **[5 each]**

a) State & prove Chapman - Kolmogrov equations for Markov Chain.

b) Describe Ehrenfest Chain model.

c) Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space s = {1, 2, 3, 4} and transition probability matrix P as follows :

$$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Show that state 1 and 2 are ergodic.

Q3) Attempt <u>any two</u> of the following: **[5 each]**

a) Explain the periodicity of a state of a Markov Chain with the help of a suitable example.

Check whether state 1 is aperiodic for the following transition probability matrix (P) of a Markov Chain with state space s = {1, 2, 3}

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

b) Let $\{X_n, n \geq 0\}$ be a Markov Chain with state space s = {0, 1, 2, 3, 4} and transition probability matrix is

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Show that state 4 is transient.

c) Explain Gambler's ruin problem.

Q4) Attempt <u>any one</u> of the following:

a) i) A Particle performs a random walk with absorbing barriers 0 & 4. Whenever it is at any position r $(0 < r < 4)$, it moves to r + 1 with probability p or to (r – 1) with probability q, s. t p + q = 1. But as soon as it reaches 0 or 4 it remains there itself. **[5]**

Let X_n be position of the particle after n moves.

Obtain a transition probability matrix of the Markov Chain $\{X_n\}$.

Also compute $P[X_2 = 3 / X_1 = 2]$

ii) If $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ are two independent Poisson processes with parameters λ_1 & λ_2 respectively, then show that **[5]**

$$P\left[N_1(t) = k \mid N_1(t) + N_2(t) = n\right] = \binom{n}{k} p^k q^{n-k}$$

where $p = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$ and $q = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}$

b) i) State & Explain postulates of Poisson process. **[3]**

ii) Explain closed set of states in M.C. **[2]**

iii) Let $\{X_n, n \geq 0\}$ be a Markov Chain with state space s = {0, 1, 2} with one step transition probability matrix **[5]**

$$P = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix}$$

and initial probability distribution $P\left[X_0 = i\right] = \dfrac{1}{3}$ for i = 0, 1, 2.

Obtain two step transition probability matrix

compute

1) $P[X_2 = 1 \mid X_0 = 0]$

2) $P[X_2 = 2, X_0 = 0]$