Total No. of Questions : 4]

SEAT No. :

P808

[Total No. of Pages : 2

[5315]-408
T.Y. B.Sc. (Semester - IV)
MATHEMATICS (Paper - VII)
MT - 347 (B) : Differential Geometry
(New Course 2013 Pattern)

Time : 2 Hours] *[Max. Marks : 40*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any five of the following : **[10]**

a) Find a parametrisation of the level curve $y^2 - x^2 = 1$.

b) Define curvature of a curve in R^3.

c) Show that the curve $\overline{r}(t) = \left(\frac{1+t^2}{t}, t+1, \frac{1-t}{t}\right)$ is a plane curve.

d) State frenet servet equation.

e) Show that second fundamental form of a plane is zero.

f) State isoperimetric inequality.

g) Define geodesics.

Q2) Attempt any two of the following : **[10]**

a) Find the equation of the tangent plane of the following surface patch at (1,0,1)

$\sigma(r,\theta) = (r \cosh\theta, r \sinh\theta, r^2)$

b) Show that the quadric $x^2 + 2y^2 + 6x - 4y + 3z = 7$ is a smooth surface with an atlas consisting of the single surface patch.

c) Find the torsion of the circular helix $r(\theta) = (a\cos\theta, a \sin\theta, b\theta)$.

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Q3) Attempt any two of the following : **[10]**

a) State and prove meusniers theorem

b) Find the area of the interior of the elipse $\gamma(t) = (a\cos t, b\sin t)$, where a and b are positive constants.

c) Find the arc-length along the cycloid $\gamma(t) = a(t - \sin t, 1 - \cos t)$, $o \in t \leq 2\pi$.

Q4) Attempt any one of the following : **[10]**

a) i) Show that every isometry is a conformal map. Give an example of a cenformal map that is not an isometry.

ii) Define a reparametrization of a parametrized curve $\gamma : (\alpha, \beta) \to \mathbb{R}^n$. When you say that a point $\gamma(t)$ of the curve γ is a regular point. Prove that any reparametrization of a regular curve is regular.

b) i) With usual notation, show that

$\|\sigma_u \times \sigma_v\| = (EG - F^2)^{1/2}$.

ii) Prove that transition maps of a smooth surface are smooth.

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