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SEAT No. :

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S.Y. B.Sc.

STATISTICS

ST-222: Sampling Distributions and Inference

(2013 Pattern) (Semester- II) (Paper-II)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Use of calculator and statistical tables is allowed.***
4) ***Symbols and abbreviations have their usual meaning.***

Q1) Attempt each of the following:

a) Choose the correct alternative in each of the following: **[1 each]**

i) If X_1 and X_2 are independent random variables (r.v.s) having N (0,1) and N (0,16) distribution respectively then probability distribution of $X_1^2+\frac{1}{4}X_2^2$ is

A) N (0,17) B) χ_2^2

C) t_2 D) $F_{1,1}$

ii) If a r.v. has chi-square distribution with variance equal to 8 then it's moment generating function (m.g.f.) is given by

A) $(1-2t)^{-2}$ B) $(1-t)^{-2}$

C) $(1-2t)^2$ D) $(1-t)^2$

iii) Suppose e_1, e_2, e_3 and e_4 are expected frequencies such that $e_1, e_2 > 5$ and $e_3+e_4 = 8$ are obtained after fitting a probability distribution in which one parameter is estimated. Then under H_0 : fitting of probability distribution is good, the test statistic used

A) χ^2 with 1 degrees of freedom (d.f.)

B) χ^2 with 2 d.f.

C) χ^2 with 4 d.f.

D) χ^2 with 3 d.f.

P.T.O.

b) State whether each of the following statement is true or false: **[1 each]**

i) If r.v. F follows $F_{2,2}$ distribution with $Q_1 = 5$ then $Q_3 = \frac{1}{5}$.

ii) Let X be a r.v. having t- distribution with 5 d.f. Then the value of μ_4 is equal to $\frac{25}{2}$.

iii) Let $X_1, X_2, \ldots, X_n$ be a random sample (r.s.) from N (μ, σ^2), μ is unknown. To test $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 > \sigma_0^2$ the rejection region is $\chi^2_{n-1} \leq \chi^2_{n-1,1-\alpha}$ at α level of significance (l.o.s.).

c) State limiting behaviour of χ^2_n as $n \to \infty$ according to Fisher's approximation. **[1]**

d) State the standard error of the statistic $\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n}$ **[1]**

e) Give one real life situation where chi-square test of independence can be used. **[1]**

f) Distinguish between two sample t-test and paired t-test. **[1]**

Q2) Attempt ANY TWO of the following: **[5 each]**

a) Find the mode of a chi-square distribution with n d.f. Also if $X \sim \chi^2_n$ and mode of the distribution is 5 find P (X > 2.167).

b) If a r.v. $U \sim N(0,1)$, $V \sim \chi^2_n$ and are independent then find the distribution of $\frac{U}{\sqrt{\frac{V}{n}}}$.

c) If $X \sim \chi^2_{10}, Y \sim \chi^2_9$ and are independent r.v.s then find

i) P [12.242 < Y < 21.666]

ii) Median of Y

iii) P[X+Y $\geq$ 21.689].

Q3) Attempt ANY TWO of the following: **[5 each]**

a) If a r.v. $X \sim F_{n_1,n_2}$ then find the distribution of $\frac{1}{X}$.

b) Derive a test statistic to test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$. Also state the assumptions if any.

c) If $\overline{X}$ and S^2 are the mean and the variance of a r.s. of size 10 from N(4,16) then find P(–1<X<4, 6.6688 < S^2 < 17.0496).

Q4) Attempt ANY ONE of the following:

a) i) If $X_1,....,X_n$ is a r.s. from N(μ, σ^2) distribution then show that sample mean ($\overline{X}$) and sample variance (S^2) are independently distributed. **[6]**

ii) For two independent normal populations we have the following information :

Sample means	$\overline{X}$=10	$\overline{Y}$=12
Sample variances	S_1^2=46	S_2^2=50
Sample sizes	n_1=15	n_2=15

To test H_0: $\mu_1=\mu_2$ against H_1: $\mu_1 \neq \mu_2$, calculate the 95% confidence interval for ($\mu_1-\mu_2$). Also give the conclusion. (Use α=5%) **[4]**

b) i) If $X \sim F_{n_1,n_2}$ and $Y \sim F_{n_2,n_1}$ then show that $P(X \geq a) + P\left(Y \geq \frac{1}{a}\right) = 1$ **[2]**

ii) A certain stimulus is administered to each of 12 patients resulted in the following increase in blood pressure:

5,2,8,–1,3,0,4,6,–2,1,5,0

Can it be concluded that the administration of the stimulus in general will be accompanied by increase in the b.p.? Use appropriate test to give the answer. (Use l.o.s.= 0.05) **[4]**

iii) Write a short note on McNemar's test. **[4]**