Total No. of Questions :5]

SEAT No. :

P345

[Total No. of Pages :4

[5215] - 2

F.Y.B.Sc.

MATHEMATICS

MT-102: Calculus and Differential Equations

(2013 Pattern) (Paper - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to right indicate full marks.*

Q1) Attempt any eight of the following: **[16]**

a) Find supremum and infimum of set

$S = \left\{1 - \frac{1}{n} : n \in N\right\}$, if they exist.

b) Let $f(x) = \frac{|x-1|}{x-1}, x \neq 1$. Find $\lim_{x \to 1^+} f(x)$.

c) By using L'Hospital's rule evaluate $\lim_{x \to 1} \frac{2^x - 2}{x-1}$.

d) State Taylor's theorem with Lagrange's form of remainder.

e) State Leibnitz's theorem.

f) Evaluate $\int_0^{\pi/2} \sin x^5 dx$.

P.T.O.

g) Define linear differential equation.

h) Examine exactness of differential equation

$$(ax+hy)dx+(hx+by)dy=0.$$

i) Solve: $(1+y^2)dx = x^2 dy$.

j) Find the orthogonal trajectories of family of circles whose centres at the origin and radius a.

Q2) Attempt any four of the following: **[16]**

a) For $x, y \in R$, prove that $|x+y| \le |x|+|y|$. Hence prove that $|x-y| \le |x|+|y|$.

b) If $\lim_{x\to a} f(x)$ exists, then prove that f is bounded in some deleted neighbourhood of the point $x = a$.

c) Evaluate: $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$.

d) State and prove Lagrange's mean value theorem.

e) If $y=\log\left(x+\sqrt{1+x^2}\right)$, then prove that

$$(1+x^2)y_{n+2}+(2n+1)x\,y_{n+1}+n^2 y_n=0.$$

f) Express the polynomial $2x^3+7x^2+x-6$ in powers of $(x-2)$.

Q3) Attempt any two of the following: **[16]**

a) i) Solve the inequality $|3x+4|<|x+2|$.

ii) Define absolute value of the real number. If $a \ge 0$, then prove that $|x| \le a$ if and only if $-a \le x \le a$.

b) i) Using $\in -\delta$ definition of limit show that $\lim_{x\to 0}\frac{2x^2+3}{x+5}=\frac{3}{5}$.

ii) Find numbers α and β if the function f is continuous at every point in (–2, 2), where

$$f(x)=\begin{cases} x+\alpha, \text{if } -2<x<0 \\ 2x+1, \text{if } 0\le x<1 \\ \beta-x, \text{if } 1\le x<2 \end{cases}$$

c) i) In Cauchy mean value theorem for functions $f(x)=e^{x}$ and $g(x)=e^{-x}$ on [a, b], show that 'c' is arithmetic mean between a and b.

ii) Suppose $a<b$ and f is derivable on (a, b). If $f'(x)>0, \forall x\in(a,b)$, then prove that f is strictly increasing function on (a, b).

Q4) Attempt any four of the following: **[16]**

a) Evaluate $\int\frac{x^2+x+1}{(x+1)^2(x+2)}dx$.

b) Explain the method of solving the differential equation $\frac{dy}{dx}=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$, when $\frac{a_1}{a_2}=\frac{b_1}{b_2}$.

c) Solve: $(x^2+y^2+x)dx+xy\,dy=0$.

d) Solve: $\frac{dy}{dx}+\frac{y}{x}=x^2y^6$.

e) Show that the family $y^2=4a(x+a)$ is self orthogonal.

f) Explain the method of solving differential equation $f(x,y,p)=0$, which is solvable for y.

Q5) Attempt any two of the following: **[16]**

a) If $I_n = \int \sin^n x\, dx, n \geq 2,$ then prove that $I_n = \dfrac{-\cos x \sin^{n-1} x}{n} + \dfrac{n-1}{n} I_{n-2}$.

Hence evaluate $\int_0^{\pi/2} \sin^8 x\, dx$.

b) i) Solve: $(x - y - z)dx - (2x - 2y - 3)dy = 0$.

ii) If M and N are homogeneous functions of same degree in x and y and $Mx + Ny \neq 0$. Then prove that $\dfrac{1}{Mx + Ny}$ is an integrating factor of differential equation $Mdx + Ndy = 0$.

c) i) Obtain the differential equation for the circuit involving L and R along with $e(t)$ all in series and solve it.

ii) Solve: $x^2 p^2 - 5xyp + 6y^2 = 0$.

$\mathcal{E}\ \mathcal{E}\ \mathcal{E}$