Total No. of Questions :5]

SEAT No. :

P344

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F.Y.B.Sc.

MATHEMATICS

MT - 101 : Algebra and Geometry

(2013 Pattern) (Paper - I)

Time : 3 Hours] ***[Max. Marks :80***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any eight of the following : **[16]**

a) Let X = {a, b, c} and R = {(a, a), (b, b), (a, b), (a, c)} be a relation on X. Is R an equivalence relation? Justify your answer.

b) Let a, b, c ∈ Z. If a|b and b|c, then prove that a|c.

c) Use remainder theorem to compute the remainder when $f(x) = x^4 - 3x^3 - 7x^2 - 2$ is divided by $g(x) = x - 2$.

d) Find the eigen values of a matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

e) If A is non-singular matrix and λ is an eigen value of A, then prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

f) Determine the nature of conic $5x^2 + 6xy + 5y^2 - 10x - 6y - 3 = 0$.

g) Find the angle between the planes $2x - y + 2z + 1 = 0$ and $3x + 2y + 6z - 5 = 0$

h) Find the equations of a line joining the points (–2, 1, 3) and (3, 1, –2)

i) Find the equation of tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at a point (1, 2, 3) on it.

j) Define right circular cone.

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Q2) Attempt any four of the following : **[16]**

a) Prove that congruence relation modulo m in Z is an equivalence relation.

b) Prove that $\sqrt{3}$ is not a rational number.

c) Prove that $(x - \alpha)$ is a factor of polynomial $f(x) \in \mathrm{R}[x]$ if and only if $f(\alpha) = 0$.

d) Using Gauss elimination method solve :

$$x + y + 2z = 8$$

$$-x - 2y + 3z = 1$$

$$3x - 7y + 4z = 10$$

e) Verify Cayley-Hamiltonian theorem for the matrix $\mathrm{A} = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$ and hence find A^{-1}.

f) Find the rank of matrix $\mathrm{A} = \begin{bmatrix} 3 & 2 & -1 & 3 \\ 2 & 3 & -4 & 7 \\ 5 & -2 & -1 & -2 \end{bmatrix}$

Q3) Attempt any two of the following : **[16]**

a) State and prove division algorithm theorem for Z.

b) i) Find the greatest common divisor of $f(x) = x^4 + 3x^2 + 2$ and $g(x) = x^3 - x^2 + x - 1$.

ii) Find the greatest common divisor of 595 and 252. Also find m and n such that (595, 252) = 595m + 252n, for some $m, n \in \mathrm{Z}$.

c) Determine the values of k so that the system

$$x + y + kz = 1$$

$$x + ky + z = 1$$

$$kx + y + z = 1$$

i) has unique solution

ii) has no solution

iii) infinite number of solutions

Q4) Attempt any Four of the following : **[16]**

a) Shift the origin to a suitable point so that the equation $x^2 - 6x - 4y - 1 = 0$ will be in the form $x^2 = 4by$. State value of b.

b) Obtain the equation of a plane in the normal form.

c) Find the equations of a line through (–2, 3, 4) and parallel to the planes $2x + 3y + 4z = 5$ and $3x + 4y + 5z = 6$.

d) Prove that the plane section of a sphere is a circle.

e) Prove that the straight line $\frac{x+1}{4} = \frac{y-2}{1} = \frac{z-2}{1}$ touches the sphere $x^2 + y^2 + z^2 = 9$. Also find the point of contact.

f) Find the equation of a cylinder whose generators are parallel to the line $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$ and z= 0.

Q5) Attempt any two of the following : **[16]**

a) Reduce the equation $5x^2 - 6xy + 5y^2 + 18x - 14y + 9 = 0$ to standard form and name the conic.

b) i) Find the angle between two lines whose d.c.s. are l_1, m_1, n_1 and l_2, m_2, n_2.

ii) Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$

c) i) Show that the spheres $x^2 + y^2 + z^2 - 4x - 2y - 4z + 5 = 0$ and $x^2 + y^2 + z^2 - 6x - 6y + 17 = 0$ touches each other. Also find point of touching.

ii) Find the equation of right circular cone having vertex at the origin, the axis of line whose equations are $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and semivertical angle is α.

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