Total No. of Questions : 4]

SEAT No. : ____

P622

[Total No. of Pages : 3

[5315] - 101

S.Y.B.Sc.

MATHEMATICS

MT-211: Multivariable Calculus - I

(2013 Pattern) (Semester - I) (Paper - I)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following: **[10]**

a) State second derivative test for extream of a function of two variables.

b) Find the tangent plane to the surface $f(x, y, z) = x^2 + y^2 + z - 9$ at the point (1, 2, 4).

c) Find the slope of the tangent to the parabola at (1, 2, 5) if the plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola.

d) Find an equation for the level curve of the function $f(x, y) = \sqrt{x^2 - 1}$ that passes through the point (1, 0).

e) State Clairaut's theorem for function of two variables.

f) Give an example of a continuous function $f(x, y)$ which does not have partial derivatives of first order.

g) Evaluate $\int_0^1 \int_0^{\pi} \int_0^{\pi} y \sin z \, dx \, dy \, dz$.

P.T.O.

Q2) Attempt any two of the following: **[10]**

a) If $f(x, y)$ is continuous at $(x_0 y_0)$ then prove that the function $f(x, y_0)$ is continuous at $x = x_0$ and the function $f(x_0, y)$ is continuous at $y = y_0$, where $f(x, y_0)$ and $f(x_0, y)$ being function of one variable. Is the converse hold? Justify.

b) State and prove Taylor's theorem for the function of two variables.

c) Show that $f(x, y) = \sqrt{|xy|}$ is continuous at (0, 0) but not differentiable at (0, 0)

Q3) Attempt any two of the following: **[10]**

a) If $w = f(u, v)$ is a differentiable function of u, and v and $u = \phi(x, y)$ & $v = \psi(x, y)$ are differentiable function of x and y then prove that the composite function $w = f[\phi(x, y), \psi(x, y)] = F(x, y)$ is differentiable function of x & y.

b) Find the directional derivative of $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right) + \sqrt{3}\sin^{-1}\left(\frac{xy}{2}\right)$ at the point (1, 1) in the direction of $3\bar{i} - 2\bar{j}$.

c) Discuss the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 12x - 3y + 5.$$

Q4) Attempt any one of the following: **[10]**

a) i) Evaluate the integral $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dy\, dx$ by changing the order of integration.

ii) Find the volume of the ellipsold $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = L$ by using the spherical polar co-ordinates.

b) i) Evaluate $\iint_R \sqrt{4x^2 - y^2}\, dx\, dy$, where R is the triangle bounded by the lines $y = 0$, $y = x$ and $x = 1$.

ii) Evaluate $\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x - y}{2}\, dx\, dy$ by applying the transformation $x = u + v,\ y = 2v$.

ℰ ℰ ℰ