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|---|---|---|
| ***Total No of Questions: [12]*** | ***SEAT NO. :*** | |
| | | **[Total No. of Pages : 6 ]** |

*B.E. Mechanical (2008 Pattern)*

*Finite Element Method*

*(Elective - III) (Semester - II)*

*Time: 3 Hours* — *Max. Marks : 100*

***Instructions to the candidates:***

1) ***Answers to the two sections should be written in separate answer books.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right side indicate full marks.***
4) ***Use of Calculator is allowed.***
5) ***Assume Suitable data if necessary.***
6) ***Additional data sheet is attached for the reference.***

## SECTION I

### Unit I

| | | | |
|---|---|---|---|
| Q1) | a) | Explain how banded skyline solution method is used to solve simultaneous equations. | [8] |
| | b) | List at least 6 advantages of Finite Element Method over analytical method. Also list disadvantages or limitations of FEM. | [8] |

**OR**

| | | | |
|---|---|---|---|
| Q2) | a) | Explain the terms essential and natural boundary conditions. Give example of each. | [8] |
| | b) | Explain in detail the method of matrix partitioning and how it is used to impose boundary conditions in finite element method. | [8] |

### Unit II

Q3) a) For the plane truss shown in Figure 3a, determine the following. Each element has E = 70 GPa, and area A = 200 $mm^2$. [10]

i. write down the elemental stiffness matrices (k) for each element,
ii. assemble k matrices to get global stiffness matrix (K),
iii. apply boundary conditions,
iv. find horizontal and vertical displacements of node 1,
v. determine reaction forces at node 2, 3 and 4.

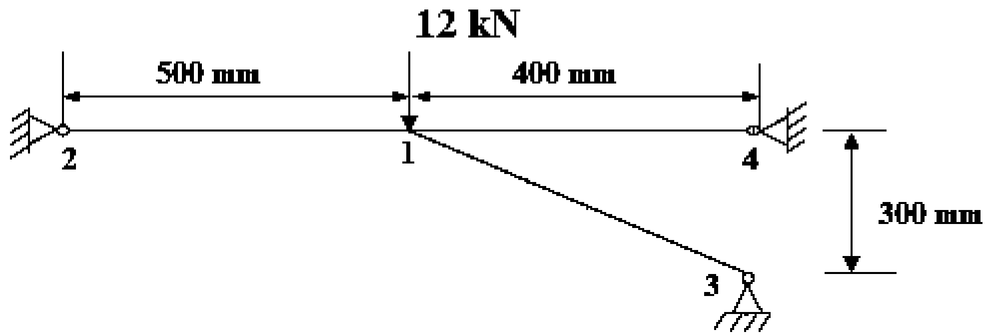


Figure 3a

b) For the five spring assemblage shown in Figure 3b, determine the displacements at nodes 2 and 3 and the reactions at nodes 1 and 4. Assume the rigid vertical bars at nodes 2 and 3 connecting the springs remain horizontal at all times but are free to slide or displace left or right. There is an applied force at node 3 of 1000 N to the right. Consider k1 = 500N/mm, k2 = k3 = 300N/mm, and k4 = k5 = 400N/mm [8]

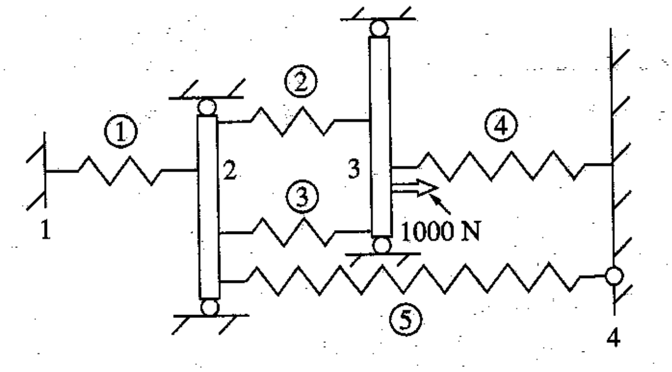


Figure 3b

**OR**

Q4) a) Derive elemental stiffness matrix and force vector for two-nodded (linear) bar element using Principle of Minimum Potential Energy (PMPE) Method. The bar element is oriented in x-direction. Assume that only concentrated forces are acting on the nodes of bar element. Ignore surface tractions and the body forces. [10]

b) For the beam shown in Figure 4b, determine the displacements and the slope at node 2. Also find the reaction forces and moments at nodes 1 and 3. Consider E = 210 GPa, and I = 4 X $10^{-4}$ $m^4$ [8]

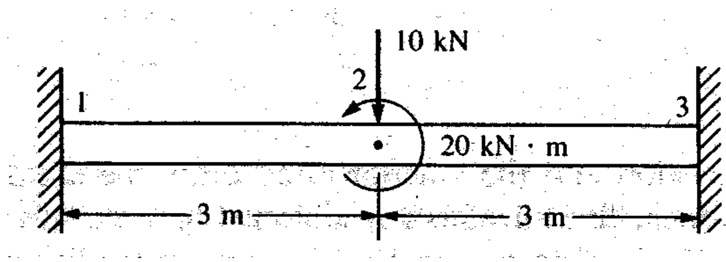


Figure 4b

**Unit III**

Q5) Evaluate the following integrals.

a) use two-point Gaussian quadrature method [5]

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

b) use two-point Gaussian quadrature method [5]

$$I = \int_1^5 [3^x + 1] dx$$

c) use three-point Gaussian quadrature method [6]

$$I = \int_4^5 \frac{2\sin(x)}{x^2} dx$$

**OR**

Q6) a) Explain with neat sketches the difference between p and h refinements in Finite Element Method. [6]

b) For the three-nodded iso-parametric bar element shown in Figure 6b, show that the Jacobean determinate is $|J| = L/2$. Also determine the shape functions $N1$ - $N3$ and the strain-displacement matrix $[B]$. Assume the displacement field as $u = a1 + a2s + a3s^2$. [10]

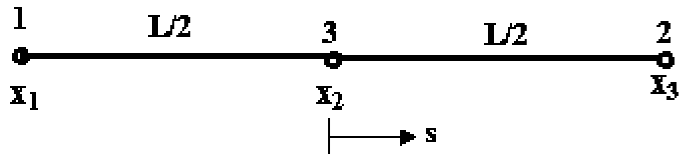


Figure 6b

**SECTION II**

**Unit IV**

Q7) Determine the temperature distribution along the length of the rod (at L/4, L/2, 3L/4, and L) as shown in Figure 7. The rod with radius of 25 mm is insulated at the perimeter. The left end has a constant temperature of 40 °C and a free stream temperature $T_\infty$ is -10 °C. Let $Kxx$ = 35 W/(m °C) and $h$ = 55 W/(m$^2$ °C). [16]

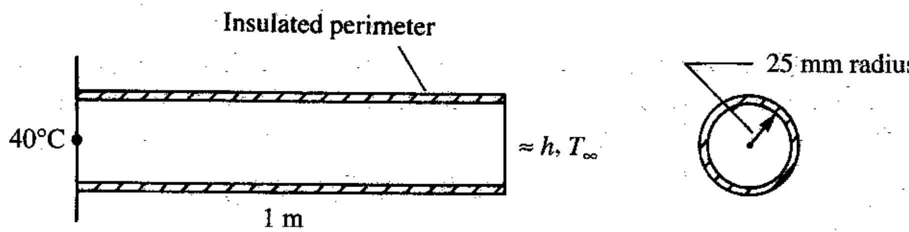


Figure 7

**OR**

Q8) A composite wall shown in Figure 8 is composed of two homogeneous slabs in contact. Let thermal conductivities be $K1$ = 1 W/(m °C) for firebrick slab 1 and $K2$ = 0.3 W/(m °C) for insulating slab 2. The left side is exposed to an ambient temperature of $T_{\infty L}$ = 1000 °C inside the furnace with heat transfer coefficient of $hL$ = 10 W/(m$^2$ °C). The right side ambient temperature is $T_\infty R$ = 25 °C outside of the furnace with heat transfer coefficient of $hR$ = 3 W/( m$^2$ °C). The thicknesses of the slabs are $L1$ = 0.20 m, and $L2$ = 0.10 m. Determine the temperature at the left edge, point between the two slabs and right edge of the composite wall. [16]

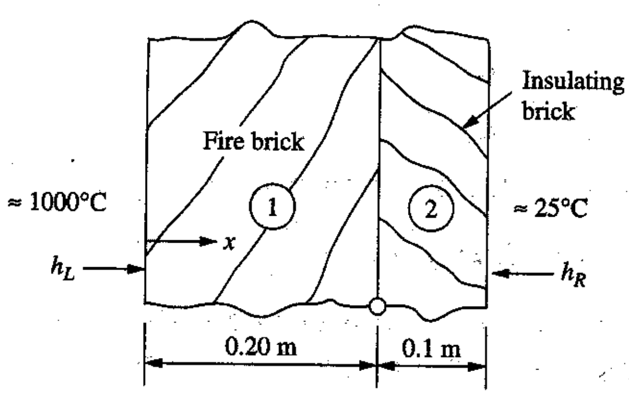


Figure 8

**Unit V**

**Q9)** a) For the stepped bar shown in Figure 9a, determine the first two natural frequencies in terms of rad/s for un-damped free vibration. Let $A_1 = A_3 = 5\ cm^2$, $A_2 = 10\ cm^2$, E = 210 GPa, and $\rho = 7860\ kg/m^3$. Use consistent mass matrices for each element. [12]

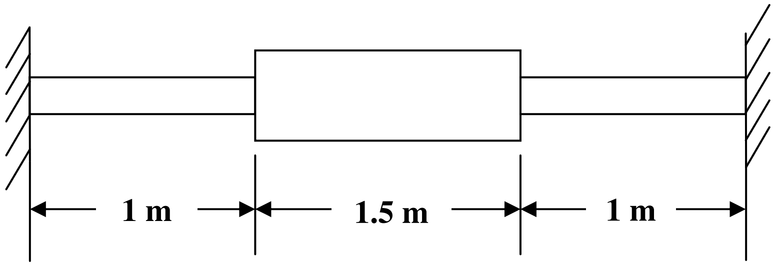


Figure 9a

b) Explain eigenvalue problem for un-damped free vibration system [6]

**OR**

**Q10)** a) For the tapered beam with square cross sectional area shown in Figure 10, set-up the problem to find characteristic equation to determine first four natural frequencies. Consider two elements of average cross sectional area and equal lengths. Each element has E = 200 GPa, density $\rho = 7500$ kg/m3, $A_1 = 9\ cm^2$, and
$A_2 = 4\ cm^2$. Use consistent mass matrix. [18]

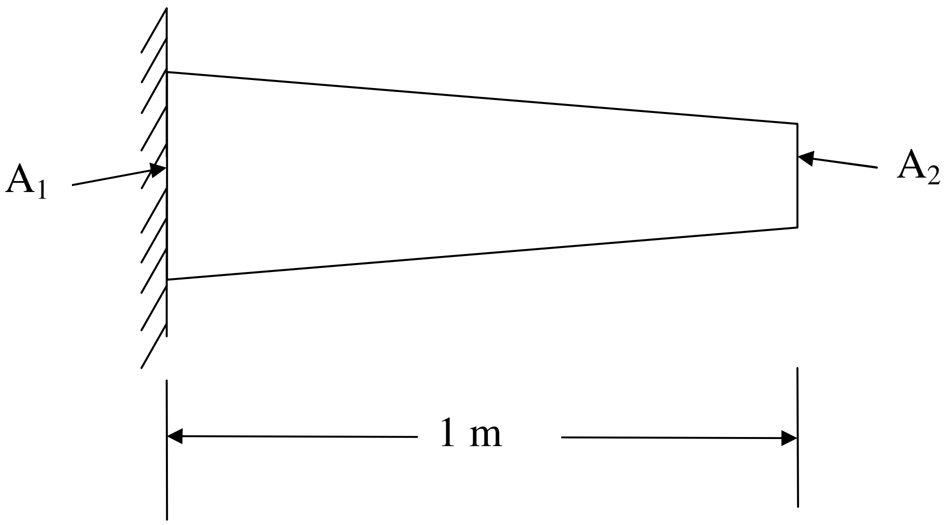


Figure 10

**Unit VI**

**Q11)** a) What are various meshing techniques? [8]

b) Explain the terms strain and stress recovery in post-processing stage. [8]

**OR**

**Q12)** Consider the truss problem shown in Figure 12 for the calculation of displacements. For this problem write down nodal coordinates, element connectivity, type of analysis, loading and boundary conditions. [16]

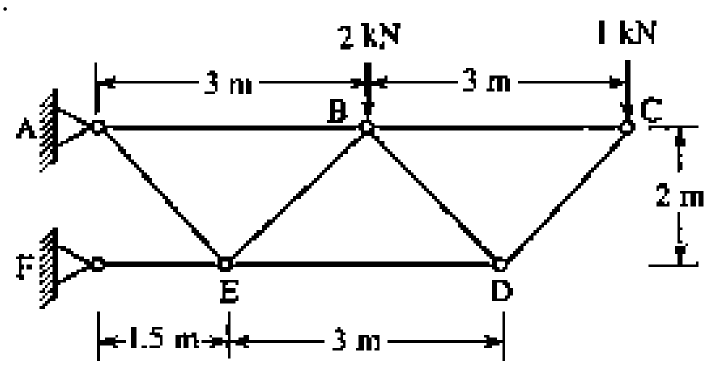


Figure 12

**DATA SHEET**

## Shape Functions:

1 Bar Element:

$$N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

2 Beam Element:

$$N_1 = \frac{1}{L^3}\left(2x^3 - 3x^2L + L^3\right)$$
$$N_2 = \frac{1}{L^3}\left(x^3L - 2x^2L^2 + xL^3\right)$$
$$N_3 = \frac{1}{L^3}\left(-2x^3 + 3x^2L\right)$$
$$N_4 = \frac{1}{L^3}\left(x^3L - x^2L^2\right)$$

## Elemental Stiffness Matrices:

1 Bar Element:

$$k_{bar} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2 Beam Element:

$$k_{beam} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3 Truss Element:

$C = \cos(\theta)$ and $S = \sin(\theta)$

$\theta$ is positive in anti clockwise direction.

$$k_{truss} = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

## Elemental Mass Matrices:

1 Bar Element:

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

(b) Lumped mass matrix:

$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2 Beam Element:

(a) Consistent mass matrix:

$$m_{consistent} = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

(b) Lumped mass matrix:

$$m_{lumped} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

## Heat Transfer Matrices:

k matrix for Conduction + Convection for bar element:

$$k = \frac{AK}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$



where, A = cross sectional area, K = Thermal Conductivity, L = Length of an element, h = Convection Coefficient, and P = Perimeter.

## Gauss Quadrature:

Table for Gauss Points for integration from -1 to 1

$$\int_{-1}^{1} y(x)dx = \sum_{i=1}^{n} W_i y_i$$

| Number of Points | Locations, $x_i$ | Associated Weights, $W_i$ |
|---|---|---|
| 1 | $x_1 = 0.000$ | 2.000 |
| 2 | $x_1, x_2 = \pm\ 0.57735$ | 1.000 |
| 3 | $x_1, x_3 = \pm\ 0.77459$<br>$x_2 = 0.000$ | $5/9 = 0.55556$<br>$8/9 = 0.88889$ |
| 4 | $x_1, x_4 = \pm\ 0.86113$<br>$x_2, x_3 = \pm\ 0.33998$ | 0.34785<br>0.65214 |