Total No. of Questions : 10] SEAT No. : [Total No. of Pages : 4]

T.E. (Common-Mechanical/Auto)

NUMERICAL METHODS AND OPTIMIZATION

(2012 Pattern) (End - Semester - II) (302047)

Time: 2½ Hours] [Max. Marks:70

Instructions to the candidates:

- 1) Answer Q.No.1 or 2, Q.No.3 or 4, Q.No.5 or 6, Q.No.7 or 8, Q.No.9 or 10.
- 2) Neat diagrams must be drawn wherever necessary.
- 3) Figures to the right side indicate full marks.
- 4) Assume suitable data, if necessary.
- **Q1)** a) Define and explain following types of errors.

[6]

- i) Truncation Error.
- ii) Round Off Error.
- iii) Absolute Error.
- iv) Relative Error.
- v) Percentage relative Error.
- vi) Inherent Error.
- b) Using Gauss Seidal method solve the following set of simultaneous equations.

$$x_1 + 20x_2 + 9x_3 = -23$$

$$2x_1 - 7x_2 - 20x_3 = -57$$

$$20x_1 + 2x_2 + 6x_3 = 28$$

Show two iterations in tabular form.

[6]

[3]

- **Q2)** a) Find the roots of $\cos x x = 0$ by Regula Falsi method. Take $x_1 = 0.6$ and $x_2 = 1$. Find the value of x for 3 iterations. [6]
 - b) Draw flow chart for Thomas algorithm method. [6]
- **Q3)** a) Maximize $Z = 2x_1 + 5x_2$ subjected to,

$$x_1 + 4x_2 \le 24$$

$$3x_1 + x_2 \le 21$$

$$x_1 + x_2 \le 9$$

$$x_1, x_2 \ge 0$$
 [5]

b) Write a note on constrained optimization.

OR

- **Q4)** a) Using Newton's method find the maximum value for the equation $x^3 5x + 3$. Take initial guess as zero up to accuracy 0.001. [5]
 - b) Write down the advantages of genetic algorithm. [3]
- Q5) a) A material is tested for cyclic fatigue failure where by a stress in MPa is applied to the material and the number of cycles needed to cause failure is measured. The results are in the table below:[8]

| N | 1 | 10 | 100 | 1000 | 10,000 | 100000 | 1000000 |
|--------|------|------|-----|------|--------|--------|---------|
| Cycles | | | | | | | |
| σ | 1131 | 1058 | 993 | 801 | 651 | 562 | 427 |

When a log-log plot of stress versus cycles is generated, the data trend shows a linear relationship. Use the method of least squares to find the equation of that straight line.

b) Find the polynomial passing through points (0, 1), (1, 1), (2, 7), (3, 25), (4, 61), (5, 12) using Newton's interpolation formula and hence find y at x = 0.5.

OR

[8]

Q6) a) The pressure (P) and volume (V) of a gas are related by the equation PV^r = K, r and K are constants. Fit this equation for the following set of observations:

| P kg/m ² | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---------------------|------|---|------|------|------|------|
| V(liters) | 1.62 | 1 | 0.75 | 0.62 | 0.52 | 0.46 |

b) A set of values of x and f(x) are given below. Using Lagrange's interpolation formula, find f(9). [8]

| X | 5 | 7 | 11 | 13 | 17 |
|--------|-----|-----|------|------|------|
| y=f(x) | 150 | 392 | 1452 | 2366 | 5202 |

Q7) a) Evaluate
$$\int_{0}^{1} \frac{\sin x}{2 + 3\sin x} dx$$
 using Simpson's 3/8th rule. Take 6 strips. [8]

b) Draw flowchart for Gauss Legendre 2 point and three point formulae combinely. [8]

OR

Q8) a) Use Trapezoidal rule to evaluate

$$\int_{0}^{1} \int_{1}^{2} \frac{2xy}{(1+x^2)(1+y^2)} dx.dy.$$

- b) Explain Simpson's $\frac{1}{3}$ rd rule graphically and derive formula for integration of a function. [8]
- **Q9)** a) The relationship between x and y is given by $\frac{dy}{dx} + xy = 2$. Estimate y at x = 5.1 using 2^{nd} order Runge-Kutta method. Assume y = 2 at x = 5. Take step size of 0.02. [10]

b) Draw flow chart for Laplace equation when plate is divided in nine parts and temperatures at four nodes are to be find out when temperatures at four sides are given. [8]

OR

- **Q10)**a) Using Runge Kutta method, solve $2\frac{d^2y}{dx^2} 3x\frac{dy}{dx} + 9y = 9$ for x = 0.1, initial conditions are x = 0, y = 1, $\frac{dy}{dx} = -2$, h = 0.1. [8]
 - b) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subjected to the conditions u(0, t) = u(1, t) = 0 and u(x, 0) = 2x for $0 \le x \le \frac{1}{2}$ and u(x, 0) = 2(1 x) for $\frac{1}{2} \le x \le 1$. Take $h = \frac{1}{4}$ and k = 1.

