

Total No. of Questions—8]

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<b>Seat No.</b>	
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**[5252]-511****S.E. (Mechanical/Mech.Sand.) (First Semester)****EXAMINATION, 2017****ENGINEERING MATHEMATICS-III****(2015 PATTERN)****Time : Two Hours****Maximum Marks : 50**

- N.B. :** (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
- (ii) Neat diagrams must be drawn wherever necessary.
- (iii) Figures to the right indicate full marks.
- (iv) Assume suitable data, if necessary.

1. (a) Solve any *two* of the following : [8]

(i)  $(D^2 - 4D + 3)y = x^3 e^{2x}$

(ii)  $(D^2 + 4)y = \sec 2x$  (using method of variation of parameter)

(iii)  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^5$ .

(b) Find Fourier sine transform of : [4]

$$\frac{e^{-ax}}{x} \text{ where } x > 0.$$

Or

2. (a) A body of weight  $W = 3\text{N}$  stretches a spring of 15 cm. If the weight is pulled down 10 cm below the equilibrium position and given a downward velocity 60 cm/sec, determine the amplitude, period and frequency of motion. [4]

P.T.O.

(b) Solve any one : [4]

(i) Find the Laplace transform of :

$$e^{-4t} \int_0^t \frac{\sin 3t}{t} dt.$$

(ii) Obtain the Inverse Laplace transform of :

$$\frac{2s + 5}{s^2 + 4s + 13}$$

(c) Using Laplace transform solve the differential equation : [4]

$$\frac{dy}{dx} + 2y(t) + \int_0^t y(t) dt = \sin t, \text{ given } y(0) = 1.$$

3. (b) The first four central moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the first four moments about the mean. Also evaluate  $\beta_1$ ,  $\beta_2$  and comment upon the skewness and kurtosis of the distribution. [4]

(b) In a certain examination test, 2000 students appeared in a subject of mathematics. Average marks obtained were 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that marks are distributed normally.

(Given :  $A(z = 2) = 0.4772$ ). [4]

(c) Find the directional derivative of  $\phi = xy^2 + yz^3$  at  $(1, -1, 1)$  along the direction normal to the surface  $x^2 + y^2 + z^2 = 9$  at  $(1, 2, 2)$ . [4]

Or

4. (a) The two regression equations of the variables  $x$  and  $y$  are  
 $x = 19.13 - 0.87y$ ,  $y = 11.64 - 0.50x$ , find  $\bar{x}$ ,  $\bar{y}$  and coefficient  
of correlation between  $x$  and  $y$ . [4]
- (b) Prove the following (any one) : [4]

$$(i) \quad \bar{b} \times \nabla[\bar{a} \cdot \nabla \log r] = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a} \cdot \bar{r})}{r^4} (\bar{b} \times \bar{r})$$

$$(ii) \quad \nabla^4(r^2 \log r) = \frac{6}{r^2}$$

- (c) Show that the vector field :

$$\bar{F} = (y^2 \cos x + z^2)\bar{i} + (2y \sin x)\bar{j} + 2xz\bar{k} \text{ is irrotational and find}$$

scalar field such that  $\bar{F} = \nabla\phi$ . [4]

5. (a) Evaluate using Green's theorem  $\int_C \bar{F} \cdot d\bar{r}$  where  $\bar{F} = x^2\bar{i} + xy\bar{j}$   
and 'C' encloses the region of first quadrant of circle  
 $x^2 + y^2 = 1$ . [4]

- (b) Use divergence theorem to evaluate  $\iiint_S \bar{F} \cdot d\bar{S}$ , where  
 $\bar{F} = y^2z^2\bar{i} + z^2x^2\bar{j} + x^2y^2\bar{k}$  and S is the upper part of the sphere  
 $x^2 + y^2 + z^2 = a^2$  above the  $xoy$  plane. [5]

- (c) Evaluate  $\iint_S (\nabla \times \bar{F}) \cdot \hat{n} dS$  for the surface of the paraboloid :

$$z = 4 - x^2 - y^2 (z \geq 0) \text{ and } \bar{F} = y^2\bar{i} + z\bar{j} + xy\bar{k}. [4]$$

Or

6. (a) Evaluate  $\int_C \bar{F} \cdot d\bar{r}$ ,  $\bar{F} = xy\bar{i} + x^2\bar{j}$ , where C is the curve  $y^2 = x$ , joining (0, 0) and (1, 1). [4]

(b) Evaluate  $\iint_S (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}) \cdot d\bar{S}$ , where S is the surface of the sphere  $x^2 + y^2 + z^2 = a^2$ . [4]

(c) Evaluate  $\iint_S (\nabla \times \bar{F}) \cdot \hat{n} dS$  for the surface of first quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and  $\bar{F} = -y^3\bar{i} + x^3\bar{j}$ . [5]

7. (a) If  $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$  represents the vibration of a string of length  $l$  fixed at both ends, find the solution with boundary conditions :

(i)  $y(0, t) = 0$

(ii)  $y(l, t) = 0$

(iii)  $\frac{\partial y}{\partial t} = 0$  at  $t = 0$

(iv)  $y(x, 0) = 3(lx - x^2)$ ,  $0 \leq x \leq l$  [7]

(b) Solve  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  if,  $u$  is finite for all  $t$

(i)  $u(0, t) = 0^\circ$

(ii)  $u(l, t) = 0$

(iii)  $u(x, 0) = 50$ ,  $0 < x < 1$  [6]

Or

8. (a) Solve the equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  with conditions :

(i)  $u(0, \infty) = 0$

(ii)  $u(0, y) = 0$

(iii)  $u(10, y) = 0$

(iv)  $u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right), 0 \leq x \leq 10$  . [6]

(b) Use Fourier sine transform to solve the equation

$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t > 0$  subject to the following conditions :

(i)  $u(0, t) = 0, t > 0$

(ii)  $u(x, 0) = e^{-x}, x > 0$

(iii)  $u$  and  $\frac{\partial u}{\partial x} \rightarrow 0$  as  $x \rightarrow \infty$  [7]