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Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

Seat No.	

[4657]-11

S.E. (Mech./Prod/Auto) (First Semester) EXAMINATION, 2014

ENGINEERING MATHEMATICS-III

(2008 Course)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. 4, Q. No. **5** *or* Q. No. **6** from Section I. Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12** from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat sketches must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $$\frac{d^3y}{dx^3} + y = \sin(2x + 3) + e^{-x} + 2^x$$

P.T.O.

(*ii*) $(D^2 + 1)y = \operatorname{cosec} x$ (By method of variation of parameter)

(*iii*) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 4y = x^2 + 7x + 9$

(*iv*) $(D^2 - 9D + 18)\, y = e^{e^{-3x}}$

(*b*) A body of weight 1 N is suspended from a spring stretches it 4 cm. If the weight is pulled down 8 cm below the equilibrium position and then released. Find the position and velocity as function of time. Also find amplitude and period. [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $(D + 1)^2 y = 2\cos x + 3e^x$

(*ii*) $(D^2 + 4)y = \sec 2x$ (By method of variation of parameter)

(*iii*) $(D^2 + 2D + 1)y = e^{-x} \cos 2x$

(*iv*) $(3x + 2)^2 \dfrac{d^2y}{dx^2} + 3(3x + 2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 1$.

(*b*) Solve : [5]

$$\frac{dx}{dt} - 3x - 6y = t^2$$

$$\frac{dy}{dt} + \frac{dx}{dt} - 3y = e^t.$$

3. (*a*) Find Laplace Transform of the following (any *two*) : [6]

(*i*) $\dfrac{1 - \cos t}{t}$

(*ii*) $\dfrac{\cos \sqrt{t}}{\sqrt{t}}$

(*iii*) $t \int_0^t e^{-3t} \sin 2t \, dt$

(*b*) $\dfrac{dy}{dt} + 2y(t) + \int_0^t y(t) = \sin t$, given that $y(0) = 1$. [5]

(*c*) Using Fourier integral representation, show that : [6]

$$\int_0^\infty \frac{\sin \pi\lambda \sin \lambda x}{1 - \lambda^2} d\lambda = \begin{cases} \dfrac{\pi}{2} \sin x & 0 \le x \le \pi \\ 0 & x \ge \pi \end{cases}$$

Or

4. (*a*) Find Inverse Laplace Transform of the following (any *two*): [6]

(*i*) $\log\left(\dfrac{s + b}{s + a}\right)$

(*ii*) $\dfrac{s}{s^2 + 4}$

(*iii*) $\dfrac{s + 7}{s^2 + 2s + 2}$.

(*b*) Solve the integral equation : [6]

$$\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1-\lambda & 0 \le \lambda \le 1 \\ 0 & \lambda \ge 1 \end{cases}$$

(*c*) Find Fourier sine transform of : [5]

$$f(x) = \frac{1}{x}.$$

5. (*a*) Solve the equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$, where $u(x, t)$ satisfies the following conditions : [8]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$ for all t

(*iii*) $u(x, 0) = x$ in $0 < x < l$

(*iv*) $u(x, \infty)$ is finite

(*b*) A string is stretched and fastened to two points distance l apart is displaced into the form $y(x, 0) = \lambda \sin \frac{2\pi x}{l}$ from which it is released at $t = 0$, find the displacement of the string at a distance x from one end.

(Use wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$) [8]

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Or

6. (*a*) Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ with conditions : [8]

(*i*) $u(x, \infty) = 0$

(*ii*) $u(0, y) = 0$

(*iii*) $u(10, y) = 0$

(*iv*) $u(x, 0) = 100 \sin\left(\frac{\pi x}{10}\right)$.

(*b*) Use Fourier Transform to solve : [8]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, \quad t > 0.$$

where $u(x, t)$ satisfies the conditions :

(*i*) $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0 \quad , t > 0$

(*ii*) $u(x, 0) = \begin{cases} x & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t) < M$.

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SECTION II

7. (*a*) Goals scored by two teams A and B in a football season are as follows : [6]

No. of Goals in a Match	**No. of Matches**	
	Team A	**Team B**
0	27	17
1	9	9
2	8	6
3	5	5
4	4	3

Find out which team is more consistent.

(*b*) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Obtain the first four central moments, mean, standard deviation and coefficients of skewness and kurtosis. [5]

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(*c*) If 10% of the rivets produced by the machine are defective, find the probability that out of 5 rivets chosen at random :

(*i*) None will be defective

(*ii*) One will be defective. [5]

Or

8. (*a*) Obtain the regression lines for the following data : [6]

x	y
6	9
2	11
10	5
4	8
8	7

(*b*) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score between 12 and 15. [5]

Given :

$A_1 = 0.8, \; z_1 = 0.2881$

$A_2 = 0.4, \; z_2 = 0.1554$

(*c*) An average box containing 15 articles is likely to have 1 defective article. Out of 6 articles chosen what is the probability that not more than 3 are defective. [5]

9. (*a*) Show that tangent at any point on the curve $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$ makes constant angle with z-axis. [5]

(*b*) Show that the vector field $\overline{F} = f(r)\overline{r}$ is always irrotational and determine $f(r)$ such that the field is solenoidal also. [6]

(*c*) Find the directional derivative of $f = x^2y + xyz + z^3$ at $(1, 2, -1)$ along normal to the surface $x^2y^3 = 4xy + y^2z$ at the point $(1, 2, 0)$. [6]

Or

10. (*a*) With usual notations show the following results (any *two*) : [8]

(*i*) $\nabla \cdot \left[r \, \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) $\nabla^2 \; [r^2 \log r] = 5 + 6 \log r$

(*iii*) For a solenoidal vector field $\bar{E}$, show that curl curl curl curl $\bar{E} = \nabla^4 \bar{E}$.

(*b*) Show that $\bar{F} = (2xz^3 + 6y)\bar{i} + (6x - 2yz)\bar{j} + (3x^2 z^2 - y^2)\bar{k}$ is irrotational. Find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) If the directional derivative of $f = ax^2 y + by^2 z + cz^2 x$ at (1, 1, 1) has a maximum magnitude 15 in the direction parallel to the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. Find a, b, c. [5]

11. (*a*) Evaluate : [5]

$$\oint_C \bar{F} \cdot d\bar{r}$$

where

$$\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$$

and C is $x^2 + y^2 = a^2, \; z = 0$.

(*b*) Apply Stokes' theorem to evaluate $\int_C 4y\,dx + 2z\,dy + 6y\,dz$ where C is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [6]

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(*c*) Evaluate : [6]

$$\iint (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}).\, d\bar{s}$$

over the surface of sphere $x^2 + y^2 + z^2 = 9$.

Or

12. (*a*) Find the work done in moving a particle once round the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ and $z = 0$ under the force field $\bar{F} = (2x - y + z)\bar{i} + (x + y - z^2)\bar{j} + (3x - 2y + 4z)\bar{k}$. [5]

(*b*) Using Green's theorem for $\bar{F} = xy\,\bar{i} + y^2\bar{j}$ over the region R enclosed by parabola $y = x^2$ and the line $y = x$ in the first quadrant, evaluate $\int xydx + y^2dy$. [6]

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(*c*) Apply Stokes' theorem to evaluate :

$$\int_C 2y(1 - x)dx + (x - x^2 - y^2)dy + (x^2 + y^2 + z^2)dz$$

over the area of the triangle ABC cut-off from the coordinate planes by the plane $2x + 2y + z = 4$. [6]

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