S.E.-Mech Sem-I
Nov-Dec 2012 2008 pattern

Total No. of Questions—12] [Total No. of Printed Pages—8+3

Seat No.	

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S.E. (Mech.) (First Semester) EXAMINATION, 2012

ENGINEERING MATHEMATICS—III

(Common to Mech. S/W-Production, Prod. S/W & Industrial)

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $\left(D^2 + 4\right) y = x^3 + x^2 + x + 1$

(*ii*) $\left(D^2 - 5D + 6\right)y = e^{-2x} \cosh 3x$

(*iii*) $\left(D^2 - 2D + 1\right)y = xe^x \cos x$

(*iv*) $\left[x^2D^2 - 2xD - 4\right]y = x^2 + 4$ where $D \equiv \dfrac{d}{dx}$.

(*b*) The equations of motion of a particle are given by :

$$\frac{dx}{dt} + my = 0,$$

$$\frac{dy}{dt} = mx,$$

where m is some constant. Find the path of the particle. [5]

Or

2. (*a*) Solve any *three* : [12]

(*i*) $\left[(2x+1)^2 D^2 - 2(2x+1)D - 12\right]y = x$

(*ii*) $\dfrac{dx}{3z - 4y} = \dfrac{dy}{4x - 2z} = \dfrac{dz}{2y - 3x}$

(*iii*) $\left(D^2 + 5D - 3\right)y = \cos^2(3x)$

(*iv*) $\left[D^3 - 5D^2 + 8D - 4\right]y = e^{3x} + 2^{3x} + 4$ where $D \equiv \dfrac{d}{dx}$.

(*b*) Solve by using method of variation of parameters :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}.$$ [5]

3. (*a*) Find Laplace Transforms of any *two* of the following : [6]

(*i*) $L\left[e^{-2t}\cos^2 t\right]$

(*ii*) $L\left[t\, e^t \sin t\right]$

(*iii*) $L\left[\frac{\sin t}{t}\right]$.

(*b*) Solve the following differential equation by using Laplace Transform method :

$$\frac{dx}{dt} + 3x(t) + 2\int_0^t x(t)\cdot dt = t$$

where $x(0) = 0$. [6]

(*c*) Find Fourier Transform of $f(x) = e^{-|x|}$. [5]

Or

4. (*a*) Obtain inverse Laplace transforms of any *two* of the following : [8]

(*i*) $\log\left(\frac{s+4}{s+8}\right)$

(*ii*) $\frac{s+1}{(s+6)^4}$

(*iii*) $\frac{3s+7}{s^2-2s-3}$.

(*b*) Evaluate the following integral by using Laplace transforms :

$$\int_0^{\infty} e^{-2t} \cos^2 t \, dt. \quad [4]$$

(*c*) Solve the integral equation :

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda}, \;\; \lambda > 0. \quad [5]$$

5. (*a*) A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form :

$$u = a \sin\left(\frac{\pi x}{L}\right)$$

from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end by using wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \quad [8]$$

(*b*) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = L$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = L$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0, find the temperature distribution $u(x, y)$. [8]

Or

6. (a) The temperature at any point of the insulated metal rod of one metre length is given by the differential equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Find $u(x, t)$ subject to the following conditions :

(i) $u(0, t) = 0$

(ii) $u(1, t) = 0$

(iii) $u(x, 0) = 50.$ [8]

(b) Use Fourier sine transform to solve the equation :

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \; t > 0$$

subject to the conditions :

(i) $u(0, t) = 0, \; t > 0$

(ii) $u(x, 0) = e^{-x}, \; x > 0$

(iii) $u \to 0, \; \frac{\partial u}{\partial x} \to 0$ as $x \to \infty.$ [8]

SECTION II

7. (a) The following are runs scored by two batsmen A and B in ten innings :

Batsman A	**Batsman B**
101	97
27	12
0	40
36	96
82	13
45	8
7	85
13	8
65	56
14	15

Who is more consistent batsman ? [5]

(b) The first four moments about working mean 3.5 of a distribution are 0.058, 0.452, 0.082, 0.5. Calculate the first four moments about mean, mean, β_1, β_2. [6]

(c) According to past record of one day internationals between India and Pakistan, India has won 15 matches and lost 10. If they decide to play a series of 6 matches now, what is the probability of India winning the series ? (Draw is ruled out.) [5]

Or

8. (a) Find the coefficient of correlation for the following data : [6]

x	y
10	18
14	12
18	24
22	6
26	30
30	36

(b) Assume that the probability of an individual coal miner being killed in a mine accident during a year is $\frac{1}{2400}$. Calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year. [5]

(c) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score between 12 and 15.

Given : $A(z = 0.8) = 0.2881$

$A(z = 0.4) = 0.1554.$ [5]

9. (a) If

$$\bar{r} = \bar{a} \sinh t + \bar{b} \cosh t$$

where $\bar{a}$ and $\bar{b}$ are constant vectors, then prove that :

(i) $\dfrac{d^2\bar{r}}{dt^2} = \bar{r}$

(ii) $\dfrac{d\bar{r}}{dt} \times \dfrac{d^2\bar{r}}{dt^2} = \bar{a} \times \bar{b}.$ [4]

(b) Find the directional derivative of $\phi = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \dfrac{\pi}{4}$. [5]

(c) Prove the following vector identities (any *two*) : [8]

(i) $\nabla^2\left(r^2 \log r\right) = 5 + 6 \log r$

(ii) $\nabla \cdot \left[r \nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+1}}$

(iii) $\nabla \times \left[\frac{\bar{a} \times \bar{r}}{r^n}\right] = \frac{(2-n)}{r^n}\bar{a} + \frac{n}{r^{n+2}}(\bar{a} \cdot \bar{r})\bar{r}.$

Or

10. (a) If directional derivative of :

$$\phi = ax^2y + by^2z + cz^2x$$

at (1, 1, 1) has maximum magnitude 15 in the direction parallel to :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Find the value of a, b, c. [6]

(b) Verify whether the vector field :

$$\bar{F} = \left(2xz^3 + 6y\right)\bar{i} + (6x - 2yz)\bar{j} + \left(3x^2z^2 - y^2\right)\bar{k}$$

is irrotational. If so, find the scalar potential ϕ such that $\bar{F} = \nabla\phi$. [6]

(c) Show that :

$$\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$$

is solenoidal field. [5]

11. (*a*) Find work done by a force field :

$$\overline{F} = (2x - y - z)\overline{i} + \left(x + y - z^2\right)\overline{j} + (3x - 2y + 4z)\overline{k}$$

in moving a particle once round the circle $x^2 + y^2 = 9$ in XY plane. [6]

(*b*) Use Stokes' theorem to evaluate :

$$\int_C \left(4y\overline{i} + 2z\overline{j} + 6y\overline{k}\right) \cdot d\overline{r}$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$. [6]

(*c*) Evaluate :

$$\iint_S \left(x^3\overline{i} + y^3\overline{j} + z^3\overline{k}\right) \cdot d\overline{S},$$

where S is the surface of the sphere :

$$x^2 + y^2 + z^2 = 16$$ [5]

Or

12. (*a*) Evaluate $\int \overline{F} \cdot d\overline{r}$ for :

$$\overline{F} = \left(5xy - 6x^2\right)\overline{i} + (2y - 4x)\overline{j}$$

along curve C : $y = x^3$ in XY plane from point (1, 1) to (2, 8). [6]

(*b*) Evaluate :

$$\iint_S (\nabla \times \bar{F}) \cdot d\bar{S}$$

where

$$\bar{F} = (2y + x)\bar{i} + (x - y)\bar{j} + (z - x)\bar{k}$$

and S is the surface of the region bounded by $x = 0$, $y = 0$ and $x + y + z = 1$ which is not included in the XY plane. [6]

(*c*) Evaluate :

$$\iint_S \bar{r} \cdot \hat{n}\, dS$$

over the surface of a sphere of radius a with centre at origin. [5]