

S.E. - Mech Sem. I

Nov-Dec-2012 2008 pattern

Total No. of Questions—12]

[Total No. of Printed Pages—8+3

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S.E. (Mech.) (First Semester) EXAMINATION, 2012

ENGINEERING MATHEMATICS—III

(Common to, Mech. S/W-Production, Prod. S/W & Industrial)

(2008 PATTERN)

Time : Three Hours

Maximum Marks : 100

N.B. :— (i) Answer *three* questions from Section I and *three* questions from Section II.

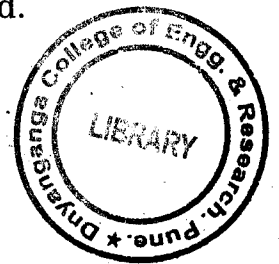
(ii) Answers to the two Sections should be written in separate answer-books.

(iii) Neat diagrams must be drawn wherever necessary.

(iv) Figures to the right indicate full marks.

(v) Use of electronic pocket calculator is allowed.

(vi) Assume suitable data, if necessary.



SECTION I

1. (a) Solve any *three* :

[12]

(i) $(D^2 + 4)y = x^3 + x^2 + x + 1$

P.T.O.

$$(ii) \quad (D^2 - 5D + 6)y = e^{-2x} \cosh 3x$$

$$(iii) \quad (D^2 - 2D + 1)y = xe^x \cos x$$

$$(iv) \quad [x^2 D^2 - 2xD - 4]y = x^2 + 4 \quad \text{where } D \equiv \frac{d}{dx}.$$

(b) The equations of motion of a particle are given by :

$$\frac{dx}{dt} + my = 0,$$

$$\frac{dy}{dt} = mx,$$

where m is some constant. Find the path of the particle. [5]

Or

2. (a) Solve any three : [12]

$$(i) \quad [(2x+1)^2 D^2 - 2(2x+1)D - 12]y = x$$

$$(ii) \quad \frac{dx}{3z-4y} = \frac{dy}{4x-2z} = \frac{dz}{2y-3x}$$

$$(iii) \quad (D^2 + 5D - 3)y = \cos^2(3x)$$

$$(iv) \quad [D^3 - 5D^2 + 8D - 4]y = e^{3x} + 2^{3x} + 4 \quad \text{where } D \equiv \frac{d}{dx}.$$

(b) Solve by using method of variation of parameters :

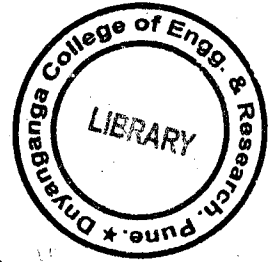
$$\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}. \quad [5]$$

3. (a) Find Laplace Transforms of any *two* of the following : [6]

(i) $L[e^{-2t} \cos^2 t]$

(ii) $L[t e^t \sin t]$

(iii) $L\left[\frac{\sin t}{t}\right]$.



(b) Solve the following differential equation by using Laplace Transform method :

$$\frac{dx}{dt} + 3x(t) + 2 \int_0^t x(t) \cdot dt = t$$

where $x(0) = 0$.

[6]

(c) Find Fourier Transform of $f(x) = e^{-|x|}$.

[5]

Or

4. (a) Obtain inverse Laplace transforms of any *two* of the following : [8]

(i) $\log\left(\frac{s+4}{s+8}\right)$

(ii) $\frac{s+1}{(s+6)^4}$

(iii) $\frac{3s+7}{s^2-2s-3}$.

- (b) Evaluate the following integral by using Laplace transforms :

$$\int_0^{\infty} e^{-2t} \cos^2 t \, dt. \quad [4]$$

- (c) Solve the integral equation :

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0. \quad [5]$$

5. (a) A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form :

$$u = a \sin\left(\frac{\pi x}{L}\right)$$

from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end by using wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}. \quad [8]$$

- (b) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = L$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = L$ and at infinity. If the edge $x = 0$ is kept at a constant temperature u_0 , find the temperature distribution $u(x, y)$. [8]

Or

6. (a) The temperature at any point of the insulated metal rod of one metre length is given by the differential equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

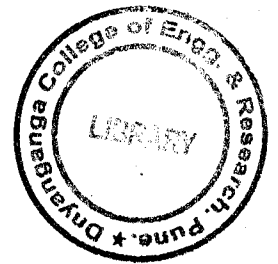
Find $u(x, t)$ subject to the following conditions :

(i) $u(0, t) = 0$

(ii) $u(1, t) = 0$

(iii) $u(x, 0) = 50.$

[8]



- (b) Use Fourier sine transform to solve the equation :

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$$

subject to the conditions :

(i) $u(0, t) = 0, \quad t > 0$

(ii) $u(x, 0) = e^{-x}, \quad x > 0$

(iii) $u \rightarrow 0, \quad \frac{\partial u}{\partial x} \rightarrow 0 \text{ as } x \rightarrow \infty.$

[8]

SECTION II

7. (a) The following are runs scored by two batsmen A and B in ten innings :

Batsman A	Batsman B
101	97
27	12
0	40
36	96
82	13
45	8
7	85
13	8
65	56
14	15

Who is more consistent batsman ? [5]

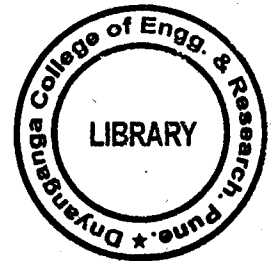
- (b) The first four moments about working mean 3.5 of a distribution are 0.058, 0.452, 0.082, 0.5. Calculate the first four moments about mean, mean, β_1 , β_2 . [6]

- (c) According to past record of one day internationals between India and Pakistan, India has won 15 matches and lost 10. If they decide to play a series of 6 matches now, what is the probability of India winning the series ? (Draw is ruled out.) [5]

Or

8. (a) Find the coefficient of correlation for the following data : [6]

x	y
10	18
14	12
18	24
22	6
26	30
30	36



- (b) Assume that the probability of an individual coal miner being killed in a mine accident during a year is $\frac{1}{2400}$. Calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year. [5]

- (c) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score between 12 and 15.

Given : $A(z = 0.8) = 0.2881$

$$A(z = 0.4) = 0.1554. \quad [5]$$

9. (a) If

$$\bar{r} = \bar{a} \sinh t + \bar{b} \cosh t$$

where \bar{a} and \bar{b} are constant vectors, then prove that :

$$(i) \quad \frac{d^2 \bar{r}}{dt^2} = \bar{r}$$

$$(ii) \quad \frac{d\bar{r}}{dt} \times \frac{d^2 \bar{r}}{dt^2} = \bar{a} \times \bar{b}. \quad [4]$$

- (b) Find the directional derivative of $\phi = e^{2x} \cos yz$ at $(0, 0, 0)$

in the direction of tangent to the curve $x = a \sin t, y = a \cos t,$

$$z = at \text{ at } t = \frac{\pi}{4}. \quad [5]$$

(c) Prove the following vector identities (any two) : [8]

$$(i) \quad \nabla^2(r^2 \log r) = 5 + 6 \log r$$

$$(ii) \quad \nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$$

$$(iii) \quad \nabla \times \left[\frac{\vec{a} \times \vec{r}}{r^n} \right] = \frac{(2-n)}{r^n} \vec{a} + \frac{n}{r^{n+2}} (\vec{a} \cdot \vec{r}) \vec{r}.$$

Or



10. (a) If directional derivative of :

$$\phi = ax^2y + by^2z + cz^2x$$

at (1, 1, 1) has maximum magnitude 15 in the direction parallel to :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Find the value of a , b , c . [6]

(b) Verify whether the vector field :

$$\vec{F} = (2xz^3 + 6y)\vec{i} + (6x - 2yz)\vec{j} + (3x^2z^2 - y^2)\vec{k}$$

is irrotational. If so, find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. [6]

(c) Show that :

$$\vec{F} = \frac{\vec{a} \times \vec{r}}{r^n}$$

is solenoidal field. [5]

11. (a) Find work done by a force field :

$$\vec{F} = (2x - y - z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$$

in moving a particle once round the circle $x^2 + y^2 = 9$ in XY plane. [6]

- (b) Use Stokes' theorem to evaluate :

$$\int_C (4y\vec{i} + 2z\vec{j} + 6y\vec{k}) \cdot d\vec{r}$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$. [6]

- (c) Evaluate :

$$\iint_S (x^3\vec{i} + y^3\vec{j} + z^3\vec{k}) \cdot d\vec{S},$$

where S is the surface of the sphere :

$$x^2 + y^2 + z^2 = 16 \quad [5]$$

Or

12. (a) Evaluate $\int \vec{F} \cdot d\vec{r}$ for :

$$\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$$

along curve C : $y = x^3$ in XY plane from point (1, 1) to (2, 8). [6]

(b) Evaluate :

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

where

$$\vec{F} = (2y + x)\vec{i} + (x - y)\vec{j} + (z - x)\vec{k}$$

and S is the surface of the region bounded by $x = 0$, $y = 0$ and $x + y + z = 1$ which is not included in the XY plane. [6]

(c) Evaluate :

$$\iint_S \vec{r} \cdot \hat{n} dS$$

over the surface of a sphere of radius a with centre at origin. [5]

