

Total No. of Questions—8]

[Total No. of Printed Pages—4+2

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[4857]-1076**S.E. (Comp./I.T.) (Second Sem.) EXAMINATION, 2015****ENGINEERING MATHEMATICS-III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50**

N.B. :— (i) Attempt *four* questions : Q. No. 1 *or* Q. No. 2, Q. No. 3 *or* Q. No. 4, Q. No. 5 *or* Q. No. 6 and Q. No. 7 *or* Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of non-programmable electronic pocket calculator is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any *two* : [8]

(i) $(D^2 - 2D - 3)y = 3e^{-3x} \sin e^{-3x} + \cos(e^{-3x})$

(ii) $(D^2 - 2D + 2)y = e^x \tan x$. (By variation of parameters)

(iii) $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2$.

(b) Find the Fourier transform of $e^{-|x|}$ and hence show that : [4]

$$\int_{-\infty}^{\infty} \frac{e^{i\lambda x}}{1 + \lambda^2} d\lambda = \pi e^{-|x|}.$$

P.T.O.

Or

- 2.** (a) An uncharged condenser of capacity C charged by applying an e.m.f. of value $E \sin \frac{t}{\sqrt{LC}}$ through the leads of inductance L and of negligible resistance. The charge Q on the plate of condenser satisfied the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}.$$

Prove that the charge at any time t is given by :

$$Q = \frac{EC}{2} \left[\sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right].$$

- (b) Find the Inverse Z-transform (any one) : [4]

(i) $F(z) = \frac{z+2}{z^2-2z+1}$ for $|z| > 1$.

(ii) $F(z) = \frac{10z}{(z-1)(z-2)}$ (Use inversion integral method).

- (c) Solve the following difference equation to find $\{f(k)\}$: [4]

$$f(k+1) + \frac{1}{4} f(k) = \left(\frac{1}{4}\right)^k, k \geq 0, f(0) = 0.$$

- 3.** (a) The first four moments of a distribution about the value 4 are $-1.5, 17, -30$ and 108 . Obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

- (b) If the probability that a concrete cube fails is 0.001. Determine the probability that out of 1000 cubes : [4]

(i) exactly two

(ii) more than one cubes will fail.

- (c) Show that : [4]

$$\vec{F} = (y \sin z - \sin x)\vec{i} + (x \sin z + 2yz)\vec{j} + (xy \cos z + y^2)\vec{k}$$

is irrotational and hence find scalar function ϕ s.t. $\vec{F} = \nabla\phi$.

Or

4. (a) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along a line equally inclined with co-ordinate axes. [4]

- (b) For a solenoidal vector field \vec{F} , show that : [4]

$$\text{curl curl curl curl } \vec{F} = \nabla^4 \vec{F}.$$

- (c) The regression equations are :

$$8x - 10y + 66 = 0 \quad \text{and} \quad 40x - 18y = 214.$$

The value of variance of x is 9. Find :

(i) The mean values of x and y

(ii) The correlation coefficient between x and y

(iii) The standard deviation of y . [4]

5. (a) Find the work done in moving a particle once round the ellipse : [4]

$$\frac{x^2}{16} + \frac{y^2}{4} = 1, \quad z = 0$$

under the field of force given by :

$$\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}.$$

- (b) Evaluate : [4]

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n} \, dS$$

where $\vec{F} = (x^3 - y^3)\vec{i} - xyz\vec{j} + y^2\vec{k}$

and S is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

- (c) Evaluate : [5]

$$\iint_S \vec{F} \cdot d\vec{S}$$

using divergence theorem, where

$$\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$$

and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$.

Or

6. (a) If [4]

$$\vec{F} = x^2\vec{i} + (x - y)\vec{j} + (y + z)\vec{k}$$

displaces a particle from A(1, 0, 1) to B(2, 1, 2) along the straight line AB, find work done.

(b) Evaluate : [4]

$$\int_C (e^x dx + 2y dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$.

(c) Evaluate : [5]

$$\iint_S \vec{F} \cdot \vec{dS}$$

using Gauss divergence theorem, where :

$$\vec{F} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$$

and S is the region bounded by :

$$x = 0, y = 0, z = 0, y = 3, x + 2z = 6.$$

7. (a) Show that $u = y^3 - 3x^2y$ is harmonic function. Find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z . [5]

(b) Using Cauchy's integral formula, evaluate : [4]

$$\int_C \frac{2z^2 + z + 5}{(z - 3/2)^2} dz$$

where C is $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

(c) Find the bilinear transformation which maps the points $z = 1, i, -1$, onto the points $w = 0, 1, \infty$. [4]

Or

8. (a) If $f(z)$ is an analytic function $v^2 = u$, then show that $f(z)$ is constant function. [4]

(b) Using residue theorem evaluate : [5]

$$\int_C \frac{z}{z^4 + 13z^2 + 36} dz$$

where 'C' is the circle $|z| = \frac{5}{2}$.

(c) Find the map of the circle $|z - i| = 1$ under the transformation $w = \frac{1}{z}$ into w -plane. [4]

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