Total No. of Questions—12] [Total No. of Printed Pages—8

Seat No.	

[4957]-210

S.E. (Electrical/Inst./Comp./I.T.) (I Sem.) EXAMINATION, 2016

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

N.B. **:—** (*i*) In Section I, solve Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**. In Section II, solve Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) Neat diagrams must be drawn wherever necessary.

(*vi*) Use of non-programmable electronic pocket calculator is allowed.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - 4D + 3)y = x^3e^{2x}$

(*ii*) $(D^2 + D + 1)y = x \sin x$

(*iii*) $(D^2 - 2D + 2)y = e^x \tan x$ (by using method of variation of parameters)

(*iv*) $(1 + x)^2 \dfrac{d^2y}{dx^2} + (1 + x) \dfrac{dy}{dx} + y = 2 \sin[\log(1 + x)]$

(*b*) Solve : [5]

$$\frac{dx}{dt} + 2x - 3y = t,$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}.$$

Or

2. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 + 2D + 1)y = xe^{-x} \cos x$

(*ii*) $(D^2 + 1)y = x^2 \sin 2x$

(*iii*) $(D^2 + 9)y = \dfrac{1}{1 + \sin 3x}$ (by using method of variation of parameters)

(*iv*) $\dfrac{dx}{3z - 4y} = \dfrac{dy}{4x - 2z} = \dfrac{dz}{2y - 3x}$.

(*b*) An electric current consists of an inductance 0.1 henry, a resistance R of 20 Ω and a condenser of a capacitance 25 microfarads. If the differential equation of electric circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

then find the charge q and current i at any time t, given that at $t = 0$, $q = 0.05$ coulombs, $i = \dfrac{dq}{dt} = 0$ when $t = 0$. [5]

3. (*a*) If $v = 3x^2y - y^3$, find its harmonic conjugate u. Find $f(z) = u + iv$ in terms of z. [5]

(*b*) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find the centre of the ellipse and its semi-major and minor axes. [5]

(*c*) Evaluate :

$$\oint_C \frac{4z^2 + z}{(z-1)^2}\, dz$$

where 'C' is the contour $|z - 1| = 2$. [6]

Or

4. (*a*) Determine k such that the function :

$$f(z) = e^x \cos y + ie^x \sin ky$$

is analytic. [5]

(*b*) Find the bilinear transformation, which maps the points $z = -1, 0, 1$ onto the points $w = 0, i, 3i$. [5]

(*c*) Evaluate :

$$\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4\cos\theta}\, d\theta$$

using Cauchy's theorem. [6]

5. (*a*) Find the Fourier transform of : [6]

$$f(x) = \begin{cases} 1 - x^2 & , \quad |x| \le 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

Hence show that :

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx = \frac{-3\pi}{16}.$$

(*b*) Find the Fourier cosine transform of the function : [5]

$$f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$$

(*c*) Find *z*-transform of (any *two*) : [6]

(*i*) $f(k) = 4^k \cdot \sin(2k + 3),\ k \geq 0$

(*ii*) $f(k) = k \cdot 5^k,\ k \geq 0$

(*iii*) $f(k) = 4^k + 5^k,\ k \geq 0.$

Or

6. (*a*) Find inverse *z*-transform of (any *two*) : [8]

(*i*) $F(z) = \frac{1}{(z-2)(z-3)},\ 2 < |z| < 3$

(*ii*) $F(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)},\ |z| > \frac{1}{2}$

(*iii*) $F(z) = \frac{z(z+1)}{z^2 - 2z + 1},\ |z| > 1.$

(*b*) Solve the difference equation : [4]

$12f(k+2) - 7f(k+1) + f(k) = 0,\ k \geq 0,\ f(0) = 0,$ $f(1) = 3.$