

Total No. of Questions—12]

[Total No. of Printed Pages—8+1

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S.E. (COMP/IT/Electrical/Instru.) (II Sem.) EXAMINATION, 2010

ENGINEERING MATHEMATICS-III

(2008 PATTERN)

Time : Three Hours

Maximum Marks : 100

N.B. :— (i) In Section I attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.

(ii) In Section II attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(iii) Answers to the two Sections should be written in separate answer-books.

(iv) Neat diagrams must be drawn wherever necessary.

(v) Figures to the right indicate full marks.

(vi) Use of electronic pocket calculator is allowed.

(vii) Assume suitable data, if necessary.

SECTION I

1. (a) Solve any *three* of the following :

[12]

(i) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3e^{-3x} \sin(e^{-3x}) + \cos(e^{-3x})$

(ii) $(x + 2)^2 \frac{d^2y}{dx^2} + 3(x + 2) \frac{dy}{dx} + y = \cos \log(x + 2)$

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$$(iii) \frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = e^x \sin^2 x$$

$$(iv) \frac{d^4 y}{dx^4} - 2 \frac{d^2 y}{dx^2} - 8y = \cosh 2x + 2^{-x}.$$

- (b) An uncharged condenser of capacity C is charged by applying an e.m.f. $E \sin \left(\frac{t}{\sqrt{LC}} \right)$, through leads of self-inductance L and negligible resistance. Prove that at any time t , the charge on one of the plates is

$$\frac{EC}{2} \left\{ \sin \left(\frac{t}{\sqrt{LC}} \right) - \frac{t}{\sqrt{LC}} \cos \left(\frac{t}{\sqrt{LC}} \right) \right\}. \quad [4]$$

Or

2. (a) Solve any *three* of the following : [12]

$$(i) \quad x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = x \log x$$

$$(ii) \quad \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^{2x} \cdot \sec^2 x \quad (\text{by variation of parameters})$$

$$(iii) \quad \frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = x^2 + e^{-x} + 1$$

$$(iv) \quad \frac{xdx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}.$$

- (b) Solve the simultaneous equations : [4]

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0.$$

3. (a) Evaluate :

$$\int_C \frac{2z^3 + z + 5}{(z - 3)^3} dz,$$

where C is $\frac{x^2}{16} + \frac{y^2}{4} = 1$. [5]

- (b) If $f(z) = u(x, y) + iv(x, y)$ is analytic, find $f(z)$ if
 $u - v = x^3 + 3x^2y - 3xy^2 - y^3$. [6]
- (c) Find the map of the straight line $2y = x$ under the transformation

$$w = \frac{2z - 1}{2z + 1}. \quad [5]$$

Or

4. (a) Find the residue of

$$f(z) = \frac{z}{(z - 1)^2(z - 2)(z - 3)}$$

at its poles and hence evaluate :

$$\int_C f(z) dz,$$

where C is the circle $|z| = 4$. [5]

- (b) Show that $u(x, y) = y + e^x \cdot \cos y$ is harmonic function. Also find its harmonic conjugate. [6]

- (c) Find the bilinear transformation which maps the points 1, i , $2i$ of z -plane onto points $-2i$, 0, 1 of w -plane. [5]

5. (a) Using Fourier integral representation, show that : [7]

$$e^{-5x} \cosh 3x = \frac{10}{\pi} \int_0^{\infty} \frac{\lambda^2 + 16}{(\lambda^2 + 4)(\lambda^2 + 64)} \cos \lambda x d\lambda.$$

- (b) Solve the integral equation : [5]

$$\int_0^{\infty} f(x) \sin \lambda x dx = \begin{cases} \lambda + 1, & 0 \leq \lambda \leq 1 \\ 2, & 1 < \lambda \leq 2 \\ \lambda - 1, & 2 < \lambda \leq 4 \\ 0, & \lambda > 4 \end{cases}$$

- (c) Find the z -transform of the following (any two) : [6]

(i) $f(k) = \left(\frac{1}{4}\right)^{|k|} \forall k$

(ii) $f(k) = a^k \sin(bk + c), k \geq 0$

(iii) $f(k) = k \cdot 3^k \cdot 2^k \cdot 5^k, k \geq 0$

Or

6. (a) Find the inverse z -transform (any two) : [8]

$$(i) \quad F(z) = \frac{z^2}{z^2 - 7z + 12}, \quad |z| > 4$$

$$(ii) \quad F(z) = \left(\frac{z}{z-2} \right)^2 \quad (\text{by integral inversion method})$$

$$(iii) \quad F(z) = \frac{z+2}{z^2 - 2z + 1}, \quad |z| > 1.$$

- (b) Solve the following by z -transform

$$f(k+2) + f(k+1) + f(k) = 0, \quad f(0) = f(1) = 1, \quad k \geq 0. \quad [5]$$

- (c) Find the Fourier transform of

$$f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

Also write Fourier integral representation of $f(x)$. [5]

SECTION II

7. (a) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the moments about the mean. Also calculate coefficients of skewness and kurtosis. [8]

(b) Find the lines of regression for the following data :

x	y
10	12
14	16
19	18
26	26
30	29
34	35
39	38

and estimate y for $x = 14.5$ and x for $y = 29.5$. [9]

Or

8. (a) If 3 of 20 tubes are defective and 4 of them are randomly chosen for inspection, then what is the probability that only one of the defective tubes will be included ? [5]
- (b) A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that not more than 5 pins will be defective in a box, find the approximate probability that a box will fail to meet the guaranteed quality. Use Poisson distribution. [6]

- (c) In a certain examination test, 2000 students appeared in a subject of Mathematics. Average marks obtained were 50% with standard deviation 5%. How many students are expected to obtain more than 60% of marks, supposing that marks are distributed normally ? [$z = 2$, $A = 0.4772$]. [6]
9. (a) Find the directional derivative of $\phi = e^{2x} \cos(yz)$ at $(0, 0, 0)$ in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \frac{\pi}{4}$. [6]
- (b) For $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$ and ϕ any scalar function show that :
- (i) $\nabla \cdot (\phi\vec{u}) = \phi(\nabla \cdot \vec{u}) + \nabla\phi \cdot \vec{u}$
- (ii) $\nabla \times (\phi\vec{u}) = \phi(\nabla \times \vec{u}) + \nabla\phi \times \vec{u}$. [6]
- (c) If a particle P moves such that at any point the position vector of P is perpendicular to the velocity vector, show that path of a particle is a circle. [4]

Or

10. (a) Show that :

$$\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$$

is irrotational. Find scalar ϕ such that $\vec{F} = \nabla\phi$. [5]

(b) If

$$\nabla\phi = (y^2 + 2y + z)\bar{i} + (2xy + 2x)\bar{j} + x\bar{k},$$

find ϕ if $\phi(1, 1, 0) = 5$.

[5]

(c) Prove that :

$$(i) \quad \nabla(\bar{r} \cdot \bar{u}) = \bar{r} \times (\nabla \times \bar{u}) + (\bar{r} \cdot \nabla)\bar{u} + \bar{u}$$

$$(ii) \quad \nabla \times (\bar{r} \times \bar{u}) = \bar{r}(\nabla \cdot \bar{u}) - (\bar{r} \cdot \nabla)\bar{u} - 2\bar{u}.$$

[6]

11. (a) Verify Green's theorem for the field

$$\bar{F} = x^2\bar{i} + xy\bar{j}$$

over the region R enclosed by $y = x^2$ and line $y = x$. [5]

(b) Evaluate :

$$\iint_S 2x^2ydydz - y^2dzdx + 4xz^2dxdy$$

where S is the surface enclosing a region bounded by hemisphere

$$x^2 + y^2 + z^2 = 9 \text{ above the } xoy \text{ plane.}$$

[6]

(c) Verify Stokes' theorem for

$$\bar{F} = xy^2\bar{i} + y\bar{j} + z^2x\bar{k}$$

for the surface of rectangular lamina bounded by $x = 0$,

$$y = 0, x = 1, y = 2, z = 0.$$

[6]

Or

12. (a) Evaluate :

$$\iint_S (2xy\bar{i} + yz^2\bar{j} + xz\bar{k}) \cdot d\bar{S}$$

over the surface of the region bounded by $x = 0$, $y = 0$,
 $z = 0$ and $x + y + z = 1$. [6]

(b) If

$$\bar{E} = \nabla\phi \quad \text{and} \quad \nabla^2\phi = -4\pi\rho.$$

prove that :

$$\iint_S \bar{E} \cdot d\bar{S} = -4\pi \iiint_V \rho dV. \quad [5]$$

(c) Evaluate :

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$$\iint_S \text{curl } \bar{F} \cdot \hat{n} dS$$

for the surface of the paraboloid

$$z = 9 - (x^2 + y^2)$$

where

$$\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xz + z^2)\bar{k}. \quad [6]$$