

Nov- Dec- 2012

Total No. of Questions—12]

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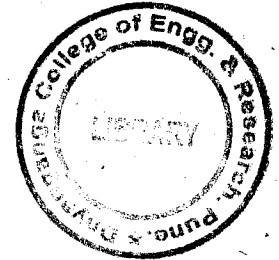
[4262]-201

S.E. (First Semester) EXAMINATION, 2012

(Common for Computer & IT)

DISCRETE STRUCTURE

(2008 PATTERN)



Time : Three Hours

Maximum Marks : 100

N.B. :— (i) Answer Q. Nos. 1 or 2, 3 or 4, 5 or 6 from Section I and Q. Nos. 7 or 8, 9 or 10 and 11 or 12 from Section II.

(ii) Answers to the two Sections should be written in separate answer-books.

(iii) Neat diagrams must be drawn wherever necessary.

(iv) Figures to the right indicate full marks.

(v) Assume suitable data, if necessary.

SECTION I

1. (a) By using mathematical induction prove that :

$$1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}.$$

[6]

P.T.O.

(b) Prove by constructing the truth table :

(i) $P \rightarrow (Q \vee R) \equiv (P \rightarrow Q) \vee (P \rightarrow R)$

(ii) $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$ is a tautology. [6]

(c) (i) Obtain the dnf. for : $\sim (p \vee q) \equiv (p \wedge q)$

(ii) Obtain the cnf : $(\sim p \wedge q \wedge r) \vee (p \wedge q)$. [6]

Or

2. (a) Among the integer 1 to 1000 :

(i) How many of them are not divisible by 3, nor by 5, nor by 7 ?

(ii) How many are not divisible by 5 and 7 but divisible by 3. [6]

(b) Prove the expression by using the Venn diagram (any two) :

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iii) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$. [6]

(c) Among 130 students, 60 study mathematics, 51 study physics and 30 study mathematics and physics. Out of 54 students studying chemistry, 26 study mathematics, 21 study physics and 12 study both mathematics and physics. All the students studying neither mathematics nor physics are studying biology.

Find :

- (i) How many are studying biology ?
- (ii) How many are studying chemistry and studying mathematics but not physics ?
- (iii) How many students are studying neither mathematics nor physics nor chemistry ? [6]

3. (a) Define the following terms with suitable example : [6]

- (i) Group
- (ii) Subgroup
- (iii) Ring
- (iv) Integral domain.

(b) Let $(A, *)$ be a group. Show that $(A, *)$ is an abelian group if and only if $a^2 * b^2 = (a * b)^2$. [6]

(c) Show that set of all idempotents in a commutative monoid S is a submonoid of S . [4]

Or

4. (a) Define the following terms with suitable example : [6]

- (i) Field
- (ii) Monoid
- (iii) Homomorphism
- (iv) Automorphism.

(b) If R is a ring such that $a^2 = a$, $\forall a \in R$, prove that : [6]

(i) $a + a = 0$, $\forall a \in R$

(ii) R is a commutative ring.

(c) Find number of code generated by the given parity check matrix

H . Also find all code words generated.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad [4]$$

5. (a) If $X = \{1, 2, 3, \dots, 7\}$ and $R = \{(x, y) | x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation. Draw the digraph of R . [4]

(b) If $S = \{1, 2, 3, \dots, 8, 9\}$. Determine whether or not each of the following is a partition of S :

(i) $\{\{1, 3, 5\}, \{2, 6\}, \{4, 8, 9\}\}$

(ii) $\{\{1, 3, 5\}, \{2, 4, 6, 8\}, \{7, 9\}\}$

(iii) $\{\{1, 3, 5\}, \{2, 4, 6, 8\}, \{5, 7, 9\}\}$

(iv) $\{\{S\}\}$

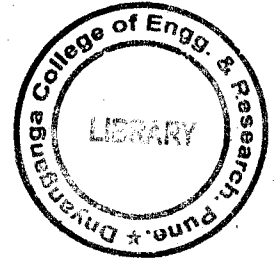
[4]

- (c) Find the transitive closure of R by Warshall's algorithm; where

$$A = \{1, 2, 3, 4, 5, 6\} \text{ and } R = \{(x, y) \mid |x - y| = 2\}. \quad [4]$$

- (d) Define the following terms :

- (i) Poset
- (ii) Equivalence class
- (iii) Chain
- (iv) Antichain.



[4]

Or

6. (a) If $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for $x \in \mathbb{R}$, where \mathbb{R} = Set of real numbers.

Find :

$$gof, fog, fof, gog, foh, fohog. \quad [6]$$

- (b) Find the homogeneous solution for the recurrence relation

$$a_n = 6a_{n-1} + 11a_{n-1} + 11a_{n-2} - 6a_{n-3}$$

$$\text{if } a_0 = 2, a_1 = 5, a_2 = 15. \quad [6]$$

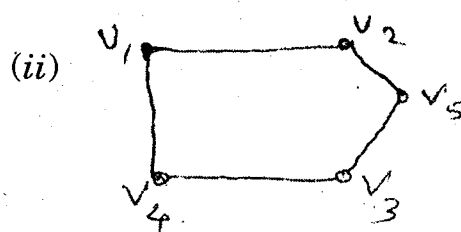
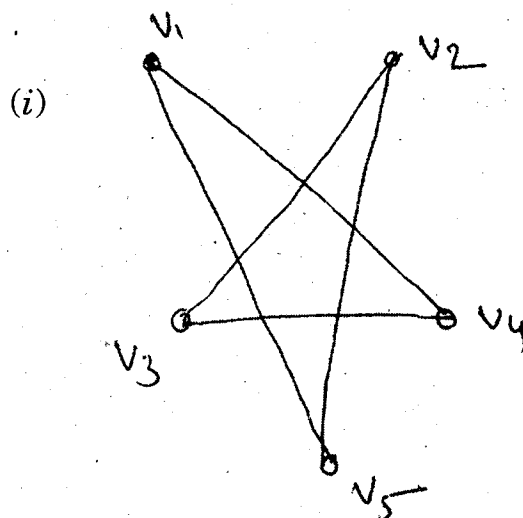
- (c) State and explain Pigeonhole principle. Show that if 7 colors are used to paint 50 bicycle, at least 8 bicycle will be of the same color. [4]

SECTION II

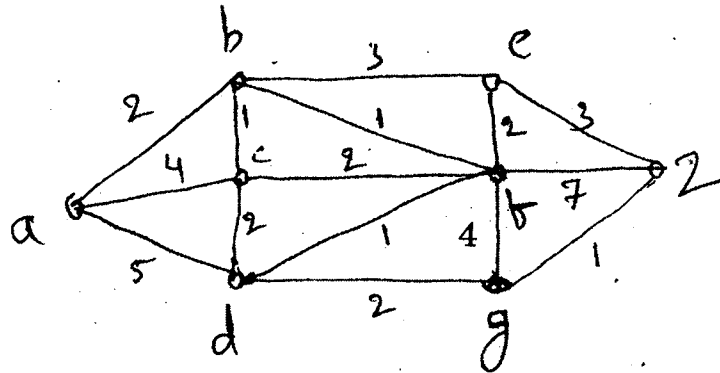
7. (a) With reference to the Graph theory define the following terms : [6]

- (i) Regular graph
- (ii) Acyclic graph
- (iii) Bipartite graph
- (iv) Multigraph.

- (b) Find the complement of the following graphs : [4]



- (c) Find the shortest path from a to z , using Dijkstra's Algorithm. [8]

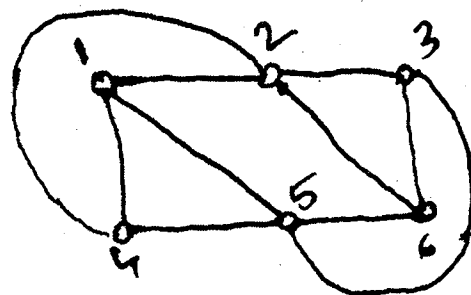
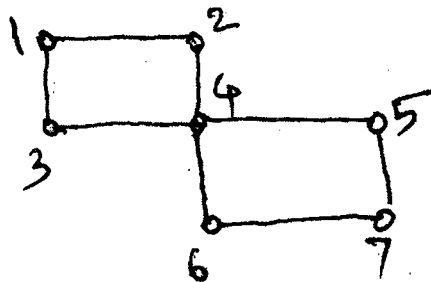


Or

8. (a) With reference to the Graph theory define the following terms : [6]

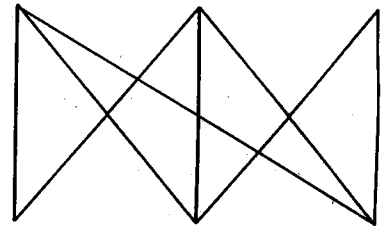
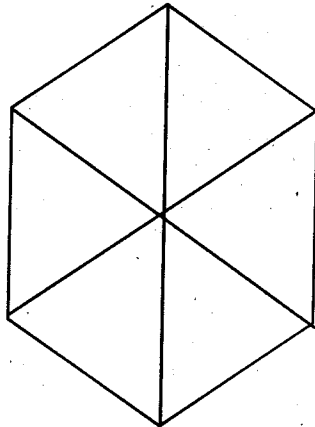
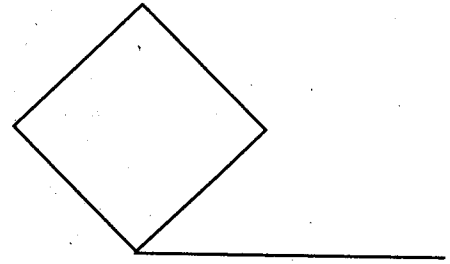
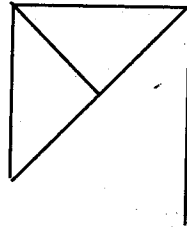
- (i) Hamilton paths and circuit
- (ii) Eulerian paths and circuit
- (iii) Planar graph
- (iv) Isomorphic graph.

- (b) (i) Determine, if the following graphs are having the Hamilton circuit or Hamilton path. Justify your answer. [4]

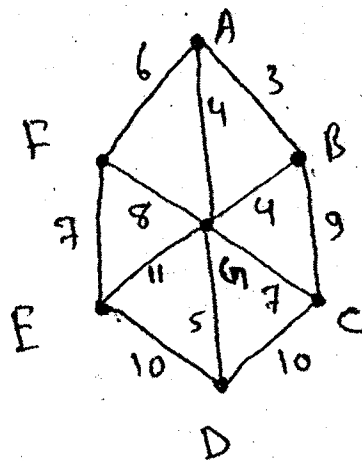


- (ii) Show that in a connected graph with 6 vertices and 12 edges each of the region is bounded by 3 edges. [4]

- (c) Identify whether the given graphs are isomorphic or not. [4]



9. (a) What do you mean by spanning tree? State the Prim's algorithm to find the minimum spanning tree. Find the minimum spanning tree of the given graph G. [8]



- (b) What is optimal binary tree ? State and explain the Huffman algorithm to find the optimal binary tree. Construct an optimal tree for the weights : 8, 9, 10, 11, 13, 15, 22. Also generate the optimal prefix code. [8]

Or

10. (a) Find the fundamental system of cutset for graph G shown in the Fig. G, with respect to spanning tree Fig T. [8]

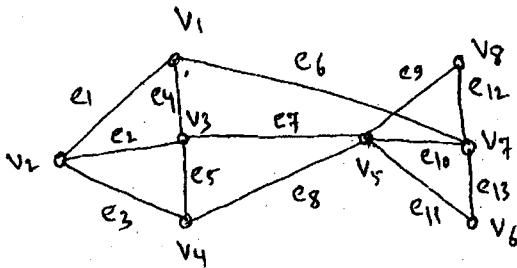


Fig. G

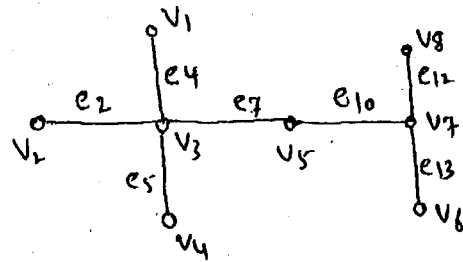
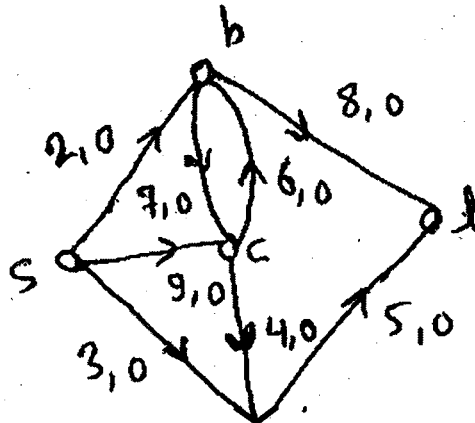
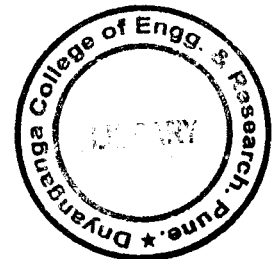


Fig. T

- (b) Determine the maximal flow in the following transport outwork. [8]



9



11. (a) A woman has 11 friends :

(i) In how many ways can she invite five of them for dinner ?

(ii) In how many ways if two of them are married and will not attend separately ? [4]

(b) (i) How many vehicle number plates can be made if each plate contains two different letters followed by three different digits ?

(ii) Find if first digit cannot be zero. [6]

(c) An 8 member team is to be formed from a group of 10 men and 15 women. In how many ways can the team be chosen if :

(i) The team must contain 4 men and 4 women

(ii) There must be more men than women

(iii) There must be at least two men ? [6]

Or

12. (a) 4 persons are chosen at random from a group of 3 men, 2 women and 4 children. Find the probability that exactly two of them will be children. [6]

- (b) A committee of 12 students consists of 3 representatives from 1st year, 4 from 2nd year and 5 from 3rd year. Out of these 12 students, 3 are to be executed from committee by drawing lots. What is the probability that :
- (i) 3 students belong to 3 different classes
 - (ii) 2 belong to one class and 1 belongs to another class
 - (iii) 3 belong to same class ? [6]
- (c) In a University 60% of professors are males and 40% are females. Also 50% of male professors and 60% female professors know computers. Find the probability that a professor knowing computer is a female. [4]

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