

Total No. of Questions—8]

[Total No. of Printed Pages—5

Seat No.	
-------------	--

[5559]-143

S.E. (E&TC/Elect.) (II Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

- N.B. :—**
- (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
 - (ii) Neat diagram must be drawn wherever necessary.
 - (iii) Figures to the right indicate full marks.
 - (iv) Use of logarithmic tables, electronic pocket calculator is allowed.
 - (v) Assume suitable data, if necessary.

1. (a) Solve the following differential equations (any two) : [8]

(i) $\frac{d^3 y}{dx^3} + 4 \frac{dy}{dx} = \sin 2x$

(ii) $\frac{d^2 y}{dx^2} - y = \frac{1}{(1 + e^{-x})^2}$ (By variation of parameter)

(iii) $(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 4 \cos (\log (1 + x))$

(b) Find the Fourier transform of a function $f(x) = e^{-|x|}$. [4]

P.T.O.

Or

2. (a) An electric circuit consist of an inductance 'L', condenser of capacity 'C' and emf $E_0 \sin \omega t$ that the charge satisfy the differential equation $\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E_0}{L} \sin \omega t$. If $\omega^2 = \frac{1}{LC}$ and initially $t = 0$, $Q = 0$ and $\dot{Q} = 0$, find the charge at any time 't'. [4]

- (b) Solve any one : [4]

(i) Find the z -transform of a function $f(k) = k^2 a^k$, $k \geq 0$.

(ii) If $f(z) = \frac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$, then find $z^{-1}(f(z))$ for $|z| > \frac{1}{4}$.

- (c) Solve the following difference equation $12f(k+2) - 7f(k+1) + f(k) = 0$; $f(0) = 0$, $f(1) = 3$, $k \geq 0$. [4]

3. (a) Find Lagrange's interpolating polynomial passing through set of points :

x	0	1	2
y	4	3	6

Use it to find y at $x = 1.5$; $\frac{dy}{dx}$ at $x = 0.5$. [4]

- (b) Use Runge-Kutta method of fourth order to obtain the numerical solutions of $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$ in the interval (1, 1.1) with $h = 0.1$. [4]

- (c) Find the directional derivative of $\phi = xy^2 + yz^3$ at $(1, -1, 1)$ along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

Or

4. (a) Show that (any one) : [4]

$$(i) \quad \nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^5} \right) = \frac{\bar{a}}{r^5} - \frac{5(\bar{a} \cdot \bar{r})\bar{r}}{r^7}$$

$$(ii) \quad \nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right] = \frac{3}{r^4}.$$

- (b) If the vector field $\bar{F} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$ is irrotational, find a, b, c and determine ϕ such that $\bar{F} = \nabla\phi$. [4]

- (c) Evaluate $\int_0^3 \frac{dx}{1+x}$ dividing the interval into 6 parts by using Simpson's $\frac{3}{8}$ th rule. [4]

5. (a) Evaluate $\int_c \bar{F} \cdot d\bar{r}$ for $\bar{F} = 2xy\bar{i} + (x^2 - y)\bar{j} + yz\bar{k}$ along a straight line joining $(0, 0, 0)$ and $(1, 2, 1)$. [4]

- (b) Use Stokes' theorem to evaluate $\int_c (2y\bar{i} + z\bar{j} + 3y\bar{k}) \cdot d\bar{r}$ where c is boundary of rectangle $0 \leq x \leq 2, 0 \leq y \leq 3, z = 1$. [4]

- (c) By using Gauss-Divergence theorem evaluate $\iiint_s (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}) \cdot d\bar{s}$ over the surface of sphere $x^2 + y^2 + z^2 = 1$. [5]

Or

6. (a) Using Green's theorem evaluate $\int_c (2 - xy)dx + y^2 dy$ over boundary of region enclosed by parabola $y^2 = x$, line $x = 1$ and x -axis in first quadrant. [4]

- (b) By using Stokes' theorem evaluate $\iint_s (\nabla \times \vec{F}) \cdot d\vec{s}$ where $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$ over surface of hemisphere $x^2 + y^2 + z^2 = a^2$ over xy -plane. [5]

- (c) By using Gauss-Divergence theorem evaluate $\iiint_s (2xy\vec{i} + yz^2\vec{j} + x^2y\vec{k}) \cdot d\vec{s}$ over total surface of region bounded by $x = 0$, $y = 0$, $z = 0$, $y = 3$ and $x + 2z = 6$. [4]

7. (a) If $f(z) = u + iv$ is an analytic function show that : [4]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2.$$

- (b) Evaluate : [5]

$$\int_c \frac{4z^2 + z}{z^2 - 1} dz \text{ where } c \text{ is } |z - 1| = \frac{1}{2}.$$

- (c) Find the bilinear transformation which maps the points $z = 2, 1, 0$ from z -plane onto the points $w = 1, 0, i$ of w -plane. [4]

Or

8. (a) If $u = 3x^2 - 3y^2 + 2y$ find v , such that the function $f(z) = u + iv$ is an analytic function. [4]
- (b) Evaluate $\int_c \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the contour $|z| = 3/2$. [5]
- (c) Find image of X-axis under the transformation $w = \frac{i-z}{i+z}$. [4]