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<b>Seat No.</b>	
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**[5252]-540****S.E. (E&TC/(Electronics Engg.) (Second Semester)****EXAMINATION, 2017****ENGINEERING MATHEMATICS-III****(2015 PATTERN)****Time : Two Hours****Maximum Marks : 50**

**N.B. :-** (i) Attempt Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6, Q. 7 or Q. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve (any two) : [8]

(i)  $(D^2 + 4D + 4)y = \sin 2x$

(ii)  $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$

(iii)  $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2$

(b) Find  $f(x)$  if  $F_c(\lambda) = e^{-3\lambda}$ ,  $\lambda > 0$ . [4]

Or

2. (a) A capacitor  $10^{-3}$  farad is in series with e.m.f. of 20 volts and an inductor of 0.4 henry, at  $t = 0$ , the charge  $Q$  and current  $I$  are zero, find  $Q$  at any time  $t$ . [4]

P.T.O.

(b) Find the inverse  $z$ -transform (any one) : [4]

(i)  $F(z) = \frac{z}{(z-1)(z-4)}, |z| > 4$

(ii)  $F(z) = \frac{1}{(z-4)(z-3)}$ .

(by inversion integral method)

(c) Solve the following difference equation to find  $f(k)$  :

$$f(k + 2) + 3f(k + 1) + 2f(k) = 0;$$

$$f(0) = 0, f(1) = 2, k \geq 0 \quad [4]$$

3. (a) Using fourth order Runge-Kutta method, solve the differential

equation :  $\frac{dy}{dx} = \frac{1}{x+y}$  with initial condition  $y(0) = 1$ .

Find  $y(0.2)$  taking  $h = 0.2$  [4]

(b) Find Lagrange's interpolating polynomial passing through set of points : [4]

$x$	2	4	5
$y$	6	20	30

(c) Find the directional derivative of  $\phi = xy^3 + yz^3$  at the point  $(2, -1, 1)$  in the direction of vector  $\bar{i} + 2\bar{j} + 2\bar{k}$ . [4]

Or

4. (a) Show that (any one) : [4]

(i)  $\nabla \cdot \left[ r \nabla \frac{1}{r^3} \right] = \frac{3}{r^4}$

$$(ii) \quad \nabla \left( \frac{\bar{a} \cdot \bar{r}}{r^5} \right) = \frac{\bar{a}}{r^5} - 5 \frac{(\bar{a} \cdot \bar{r}) \bar{r}}{r^7}$$

(b) Show that :

$$\bar{F} = (y \sin z - \sin x) \bar{i} + (x \sin z + 2yz) \bar{j} + (xy \cos z + y^2) \bar{k}$$

is irrotational. Find scalar potential  $\phi$  such that  $\bar{F} = \nabla \phi$ . [4]

(c) Stating the formula for Simpson's  $\frac{1}{3}$ rd rule, evaluate  $\int_1^{1.04} f(x) dx$  from the following data : [4]

$x$	:	1	1.01	1.02	1.03	1.04
$f(x)$	:	3.953	4.066	4.182	4.300	4.421

5. (a) Find work done by the force

$\bar{F} = (2y + 3) \bar{i} + (xz) \bar{j} + (yz - x) \bar{k}$  in taking a particle from (0,0,0) to (3, 1, 1). [4]

(b) Apply Stokes' theorem to calculate

$\int_c 4y dx + 2z dy + 6y dz$ , where  $c$  is curve of intersection of  $x^2 + y^2 + z^2 = 6z$  and  $z = x + 3$ . [5]

(c) Show that  $\bar{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \bar{A}}{\partial t}$ ,  $\bar{H} = \nabla \times \bar{A}$  are solutions of the Maxwell's equations : [4]

$$(i) \quad \nabla \times \bar{H} = \frac{1}{C} \frac{\partial \bar{E}}{\partial t}$$

$$(ii) \nabla \times \bar{E} = \frac{1}{c} \frac{\partial \bar{H}}{\partial t}, \quad \text{if}$$

$$(1) \nabla \cdot \bar{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$$

$$(2) \nabla^2 \bar{A} = \frac{1}{c^2} \frac{\partial^2 \bar{A}}{\partial t^2}$$

Or

6. (a) Using Green's lemma evaluate  $\int_c \bar{F} \cdot d\bar{r}$ , where :

$\bar{F} = \sin z \bar{i} + \cos x \bar{j} + \sin y \bar{k}$  and  $c$  is the boundary of rectangle  
 $0 \leq x \leq \pi$ ,  $0 \leq y \leq 1$  and  $z = 3$ . [4]

(b) Use divergence theorem to evaluate

$\iiint (4xz\bar{i} - y^2\bar{j} + yz\bar{k}) \cdot d\bar{s}$ , where  $s$  is the surface of the cube  
 bounded by the planes

$x = 0$ ,  $x = 2$ ,  $y = 0$ ,  $y = 2$ ,  $z = 0$ ,  $z = 2$  [5]

(c) Prove that  $\oint_c (\bar{a} \times \bar{r}) \cdot d\bar{r} = \iint_s 2\bar{a} \cdot d\bar{s}$ , where  $\bar{a}$  is a constant vector. [4]

7. (a) If  $f(z) = u + iv$  is analytic and  $u = x^2 - y^2$  find  $v$  and then  $f(z)$  in terms of  $z$ . [4]

(b) Evaluate  $\int_c \cot z dz$ , where 'C' is the circle  $|z| = 4$ . [4]

(c) Show that under the transformation  $w = z + \frac{1}{z}$ , family of circles  
 $|z| = c$  are transformed into family of ellipses in  $w$ -plane.  
 What is the transform if  $c = 1$ . [5]

Or

8. (a) If  $f(z) = u + iv$  is analytic, show that family of curves  $u = c, v = b$  are orthogonal. [4]
- (b) Evaluate  $\int_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4} dz$ , where 'C' is circle  $|z| = 2$ . [4]
- (c) Find the bilinear transformation, which maps the points  $0, -1, i$  of the  $z$ -plane on to the points  $2, \infty, \frac{1}{2}(5+i)$  of the  $w$ -plane, respectively. [5]