

Total No of Questions: [08]**SEAT NO. :****[Total No. of Pages :X]**

S.E. (ELECT/E&TC/ELEX/INSTRUCOMPUTER/IT) : 2008 Course
Engineering Mathematics-III
(Semester - I)

Time: 3 Hours**Max. Marks : 100****Instructions to the candidates:**

- 1) Answers to the two sections should be written in separate answer books.
- 2) Answer any three questions from each section.
- 3) Neat diagrams must be drawn wherever necessary.
- 4) Figures to the right side indicate full marks.
- 5) Use of Calculator is allowed.
- 6) Assume Suitable data if necessary

SECTION I

Q1) a) Solve any three [12]

1. $(D^3 - 25D)y = \cos h(2x)\sinh(3x)$
2. $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$
3. $(D^2 + 1)y = \sec x \tan x$ (by Method of Variation of Parameters)
4. $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = x \cos(\log x)$

b) Solve [05]

$$4 \frac{du}{dx} + u - v = 0$$

$$2 \frac{dv}{dx} + u - v = 0$$

OR

Q2) a) Solve any Three [12]

1. $(D^2 + 5D + 4)y = 7x + 9$
2. $(x + 1)^2 \frac{d^2y}{dx^2} + (x + 1) \frac{dy}{dx} + y = \log(x + 1)$
3. $(D^2 + 2D + 5)y = \sin^2 x$
4. $(D^2 - 4D + 4)y = e^{2x} \sec^2 x$
(By Method of Variation of Parameters)

b) An unchanged condenser of capacity C charged by applying an emf of value E sin wt (where $w = \frac{1}{\sqrt{LC}}$) through inductance L and of negligible resistance. The [05]

charge q on the plate of condenser satisfies the differential equation $L \frac{d^2q}{dt^2} + \frac{q}{C} = E \sin wt$. Find charge q at any time t.

Q3) a) Evaluate $\oint_C \frac{z^3 - 5}{(z+1)^2(z-2)} dz$ [06]
Where c is the circle $|z|=3$

b) Find the bilinear transformation which maps the points 1, i, -1 of the z-plane on to the points i, 0, -i of the w-plane. [05]

c) If $u=3x^2-3y^2+2y$, find v such that $f(z) = u + iv$ is analytic and determine $f(z)$ in terms of z. [05]

OR

- Q4) a) Show that $w = \frac{z-i}{1-iz}$ maps upper half of z-plane into interior of the unit circle in w-plane. [05]
- b) Show that analytic function $f(z)$ with constant amplitude is constant. [05]
- c) Evaluate $\oint_c \frac{z^2+2}{(z+1)(z^2-9)} dz$ [06]
Where c is $|z-3|=5$

- Q5) a) Find z-Transform of the following (Any two) [06]

1. $f(k) = \cos h\left(\frac{k\pi}{2} + 6\right), k \geq 0$

2. $f(k) = ke^{-ak}, k \geq 0$

3. $f(k) = 2^k + 3^k, k \geq 0$

- b) Solve the difference equation [05]

$$f(k+2) - 3f(k+1) + 2f(k) = 0, \quad f(0) = 0, f(1) = 1$$

- c) Using Fourier Integral representation, show that [06]

$$\int_0^{\infty} \frac{\sin \lambda \pi \sin \lambda x}{1 - \lambda^2} d\lambda = \begin{cases} \frac{\pi}{2} \sin x & |x| \leq \pi \\ 0 & |x| > \pi \end{cases}$$

OR

- Q6) a) Find Fourier Transform of the following function [05]

$$f(x) = e^{-|x|}, \quad -\infty < x < \infty$$

- b) Using Inverse Fourier Cosine Transform, find f(x) [06]

$$\text{If } F_c(\lambda) = \begin{cases} a - \frac{\lambda}{2} & \lambda \leq 2a \\ 0 & \lambda > 2a \end{cases}$$

- c) Find Inverse z-Transform of the following (Any Two) [06]

1. $\frac{z^2}{(z-\frac{1}{4})(z-\frac{1}{5})}, \quad |z| > \frac{1}{4}$

2. $\frac{1}{(z-3)(z-4)}, \quad |z| < 3$

3. $\frac{z+1}{(z-1)^2} \quad (\text{By Inversion integral method})$

SECTION II

- Q7) a) Find Lagranges interpolating polynomial passing through set of points [05]

x	0	1	3
y	7	3	1

Use it to find $\frac{dy}{dx}$ at $x=2$

- b) Evaluate $\int_0^6 \frac{1}{1+x} dx$ using simpson's $\frac{1}{3}$ rule by dividing the interval into 6 equal parts. [05]

- c) Using Modified Euler's method, find an approximate value of y when $x=0.1$ and [06]

www.sppuonline.com 0.2, given that $\frac{dy}{dx} = x + y, y(0) = 1$

OR

- Q8) a) Find the value of y for x=5 for the following table of x, y values using Newton's forward difference formula [05]

x	4	6	8	10
y	1	3	8	16

- b) Distance covered and corresponding speeds of motors vehicle are as under [06]

S	0	10	20	30	40	50	60
V	21	39	62	64	72	56	45

Find the time taken to travel whole distance.

- c) Using fourth order Runge-Kutta method, evaluate the value of y when x=0.4 [05]
given that

$$\frac{dy}{dx} = \frac{1}{x+y}, y(0) = 1, h = 0.2$$

- Q9) a) Find the directional derivative of $\phi = xy^2 + yz^3$ at (2, -1, 1) in the direction of normal to surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2) [06]

- b) If $\vec{r} = \vec{a} \sinh t + \vec{b} \cosh t$ where \vec{a} and \vec{b} are constant vectors then prove that [06]

$$1. \frac{d^2 \vec{r}}{dt^2} = \vec{r}$$

$$2. \frac{d\vec{r}}{dt} \times \frac{d^2 \vec{r}}{dt^2} = \vec{a} \times \vec{b}$$

- c) For a solenoidal vector field \vec{E} , show that $\text{curl curl curl curl } \vec{E} = \nabla^4 \vec{E}$ [05]

OR

- Q10) a) If the directional derivative of [06]

$$\phi = a(x+y) + b(y+z) + c(x+z)$$

has maximum value 12 in the direction parallel to the line

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{3},$$

Find the values of a, b, c.

- b) Verify whether the vector field [05]

$$\vec{F} = (2xz^3 + 6y)\vec{i} + (6x - 2yz)\vec{j} + (3x^2z^2 - y^2)\vec{k}$$

Is irrotational, If so find corresponding scalar potential ϕ .

- c) Prove that (Any two) [06]

$$1. \nabla \times [\vec{a} \times (\vec{b} \times \vec{r})] = \vec{a} \times \vec{b}$$

$$2. \nabla \cdot \left[r \nabla \left(\frac{1}{r^3} \right) \right] = \frac{3}{r^4}$$

$$3. \nabla^2(r^2 \log r) = 5 + 6 \log r$$

- Q11) a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ along curve c: $x=t, y=t^2, z=t^3$ from (0, 0, 0) to (1, 1, 1) where [05]

$$\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$$

- b) Evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} ds$ where 's' is the curved surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z=2$ where $\vec{F} = 3(x-y)\vec{i} + 2xz\vec{j} + xy\vec{k}$ [06]

- c) Evaluate $\iint_S (x^3\vec{i} + y^3\vec{j} + z^3\vec{k}) \cdot d\vec{s}$ where 's' is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [06]

OR

- Q12) a) Using Green's Theorem evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \cos y \vec{i} + x(1 - \sin y)\vec{j}$ [05]

where c: $\frac{x^2}{25} + \frac{y^2}{9} = 1, z = 0$.

- b) Use divergence theorem to evaluate [06]

$\iint_s (y^2 z^2 \bar{i} + z^2 x^2 \bar{j} + x^2 y^2 \bar{k}) \cdot d\bar{s}$ where s is the upper part of sphere $x^2 + y^2 + z^2 = 9$.

c) Maxwell's equation are given by

[06]

$$\nabla \cdot \bar{E} = 0, \nabla \cdot \bar{H} = 0, \nabla \times \bar{E} = \frac{\partial \bar{H}}{\partial t}, \nabla \times \bar{H} = \frac{\partial \bar{E}}{\partial t}$$

Then show that \bar{E} and \bar{H} satisfy

$$\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$$