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SE - E&TC Sem - I

Total No. of Questions—12] [Total No. of Printed Pages—8

[3962]-156

S.E. (E&TC/Electronics) (II Sem.) EXAMINATION, 2011

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) From Section I, attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

From Section II, attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

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SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 4D + 3)\, y = e^x \cos 2x$

(*ii*) $(D^2 - 4D + 4)\, y = e^{2x} + x^2$

(*iii*) $\dfrac{d^2y}{dx^2} + y = \tan x$ [By method of variation of parameters]

(*iv*) $x^2 \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} - 3y = x^2 \log x.$

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(*b*) Solve :

$$\frac{dx}{dt} - 5y = 4\cos 2t$$

$$\frac{dy}{dt} + 5x = 4\sin 2t.$$ [5]

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 1)\, y = e^{-x} \sin e^{-x} + \cos e^{-x}$

(*ii*) $(D^2 - 4)\, y = x \sinh x$

(*iii*) $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} - y = \log(x+1)^2 - 1$

(*iv*) $\dfrac{dx}{2x} = \dfrac{dy}{-y} = \dfrac{dz}{4xy^2 - z}.$

(*b*) An electric circuit consists of an inductance 0.1 henry, a resistance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge q and current i at any time t, given that, at $t = 0$, $q = 0.05$ coulombs, $i = \dfrac{dq}{dt} = 0$ when $t = 0$. [5]

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3. (*a*) If $u + v = e^x (\cos y + \sin y)$, find analytic function $f(z) = u + iv$. [5]

(*b*) Evaluate :

$$\oint_C \frac{\sin 2z}{\left(z + \frac{\pi}{3}\right)^4} dz,$$

where C is $|z| = 2$. [6]

(*c*) Show that the map

$$w = \frac{2z + 3}{z - 4}$$

transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. [5]

Or

4. (*a*) Show that analytic function with constant magnitude is constant. [5]

(*b*) Evaluate :

$$\int_0^{2\pi} \frac{\sin^2 \theta}{5 + 4\sin\theta} d\theta$$

using Cauchy residue theorem. [6]

(*c*) Find the bilinear transformation, which maps the points $z = -1, 0, 1$ on to points $w = 0, i, 3i$. [5]

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5. (*a*) Find the z-transform of the following : [6]

(*i*) $f(k) = 2^k \; e^{-4k}$

(*ii*) $f(k) = \dfrac{e^{-k} - e^{-2k}}{k}, \; k \geq 0.$

(*b*) Find the Fourier cosine transform of the following function : [6]

$$f(x) = 1 - x^2, \quad |x| < 1$$
$$= 0, \quad |x| > 1,$$

and hence evaluate :

$$\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx.$$

(*c*) Obtain $f(k)$, given that :

$$f(k + 1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \; k \geq 0, \; f(0) = 0.$$ [5]

Or

6. (*a*) Find inverse z-transform of the following : [6]

(*i*) $\dfrac{z^2}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}, \; |z| < \dfrac{1}{5}$

(*ii*) $\dfrac{10z}{(z - 1)(z - 2)}.$

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(*b*) Find the Fourier sine and cosine transform of the function $f(x) = e^{-ax}$, $a > 0$. [6]

(*c*) Solve the integral equation :

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}, \quad \lambda > 0.$$ [5]

SECTION II

7. (*a*) For the following tabulated data :

x	y
0	1
1	5
2	25
3	100
4	250

find y at $x = 0.5$ using Newton's forward difference formula. [5]

(*b*) Evaluate numerically

$$\int_0^1 \frac{1}{1+x^2} \, dx$$

taking $h = \frac{1}{6}$ by using Simpson's $\frac{3}{8}$ rule. Compare the result with exact value. [6]

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(c) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2};$$

$y(1) = 1$ to find y at $x = 1.1$ taking $h = 0.1$. [5]

Or

8. (a) Find Lagrange's interpolating polynomial passing through set of points (0, 2), (2, –2), (3, –1). Use it to find $\frac{dy}{dx}$ at $x = 2$. [5]

(b) Evaluate numerically

$$\int_4^{5.2} \log_e x \, dx$$

taking $h = 0.2$ by using Simpson's $\frac{1}{3}$ rule. Compare the result with exact value. [6]

(c) Using modified Euler's method solve equation :

$$\frac{dy}{dx} = x - y^2;$$

$y(0) = 1$ to calculate y at $x = 0.2$ taking $h = 0.2$. [5]

9. (a) If the directional derivative of

$$\phi = axy^2 + byz + cz^2x^2$$

at (2, 1, 1) has maximum magnitude 12 in a direction parallel to x-axis, find the values of a, b, c. [5]

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(*b*) Show that :

$$\bar{F} = \left(6xy + z^3\right)\bar{i} + \left(3x^2 - z\right)\bar{j} + \left(3xz^2 - y\right)\bar{k}$$

is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) With usual notations establish : [8]

(*i*) $\bar{a} \cdot \nabla\left(\bar{b} \cdot \nabla\frac{1}{r}\right) = \frac{3(\bar{a} \cdot \bar{r})(\bar{b} \cdot \bar{r})}{r^5} - \frac{\bar{a} \cdot \bar{b}}{r^3}$

(*ii*) $\nabla^4\left(r^2 \log r\right) = \frac{6}{r^2}$.

Or

10. (*a*) Find the directional derivative of

$$\phi = e^{2x} \cos yz$$

at (0, 0, 0) in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$, at $t = \frac{\pi}{4}$. [6]

(*b*) Show that the vector field $f(r)\,\bar{r}$ is always irrotational. Determine $f(r)$ such that the field is solenoidal also. [6]

(*c*) If

$$\frac{d\bar{u}}{dt} = \bar{w} \times \bar{u}$$

and

$$\frac{d\bar{v}}{dt} = \bar{w} \times \bar{v}$$

prove that :

$$\frac{d}{dt}(\bar{u} \times \bar{v}) = \bar{w} \times (\bar{u} \times \bar{v}).$$ [5]

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11. (*a*) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r}$$

for $\bar{F} = 3x^2 \bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the path $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [6]

(*b*) Using Stokes' theorem, evaluate :

$$\int_C (ydx + zdy + xdz),$$

C being intersection of $x^2 + y^2 + z^2 = a^2$, $x + z = a$. [6]

(*c*) Prove that :

$$\iint_S (\phi\nabla\psi - \psi\nabla\phi) \cdot d\bar{s} = \iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV.$$ [5]

Or

12. (*a*) Using Green's theorem evaluate :

$$\oint_C \bar{F} \cdot d\bar{r}$$

where $\bar{F} = y^3 \bar{i} - x^3 \bar{j}$ and C is the circle $x^2 + y^2 = a^2$, $z = 0$. [6]

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(*b*) Using Gauss's divergence theorem, evaluate :

$$\iint_S \bar{F} \cdot d\bar{s}$$

where $\bar{F} = (2xy + z)\bar{i} + y^2 \bar{j} - (x + 3y)\bar{k}$ and S is the closed surface bounded by $2x + 2y + z = 6$, $x = 0$, $y = 0$, $z = 0$. [6]

(*c*) Two of Maxwell's electromagnetic equations are $\nabla \cdot \bar{B} = 0$ and $\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$. Given that $\bar{B} = \text{curl}\,\bar{A}$, then deduce that :

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\text{grad}\,V$$

where V is a scalar point function. [5]

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