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**[4957]-1035****S.E. (Electrical/Instrumentations) (First Sem.)****EXAMINATION, 2016****ENGINEERING MATHEMATICS-III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50****N.B. :— (i) Neat diagrams must be drawn wherever necessary.****(ii) Figures to the right indicate full marks.****(iii) Use of electronic pocket calculator is allowed.****(iv) Assume suitable data, if necessary.****1. (a) Solve any two differential equations : [8]**

$$(i) \frac{d^2y}{dx^2} + 4y = \sin x \cos 3x$$

$$(ii) (2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1) \frac{dy}{dx} - 12y = 6x$$

$$(iii) \frac{d^2y}{dx^2} + y = \operatorname{cosec} x \text{ by the method of variation of parameters.}$$

**(b) Solve the differential equation by using Laplace transform method : [4]**

$$\frac{d^2y}{dt^2} - 2 \frac{dy}{dt} + y = e^{-2t}, \quad y=0, \quad y' = 0 \quad \text{at } t=0.$$

P.T.O.

*Or*

2. (a) An emf applied to the circuit containing a condenser C and inductance L in series at  $t = 0$  is  $E \sin pt$ . The current  $i$  satisfies the integro differential equation :

$$L \frac{di}{dt} + \frac{1}{C} \int i \, dt = E \sin pt$$

where  $p^2 = \frac{1}{LC}$ . Find the current in the circuit at any time  $t$ .

(solve by taking  $i = -\frac{dq}{dt}$ ) [4]

- (b) Solve any one of the following : [4]

(i)  $L[t e^{3t} \cos 2t]$

(ii)  $L^{-1} \left[ \frac{2s+1}{(s+4)(s-6)} \right]$ .

- (c) Obtain  $L[f(t)]$ ,

where  $f(t) = ap, 0 < t < \frac{\pi}{p}$

$$= 0, \quad , \quad \frac{\pi}{p} < t < \frac{2\pi}{p}$$

and  $f\left(t + \frac{2\pi}{p}\right) = f(t)$ . [4]

3. (a) Find the Fourier sine transform of the function : [4]

$$F(x) = \begin{cases} x & , \quad 0 \leq x \leq 1 \\ 2-x & , \quad 1 \leq x \leq 2 \\ 0 & , \quad x > 2 \end{cases}.$$

(b) Attempt any one : [4]

(i) Find  $z$  transform of :

$$F(k) = 2^k \cos(3k + 2); \quad k \geq 0.$$

(ii) Find the inverse  $z$ -transform of :

$$\frac{3z^2 + 2z}{z^2 - 3z + 2}; \quad 1 < |z| < 2.$$

(c) If the directional derivative of  $d = axy + byz + czx$  at  $(1, 1, 1)$  has maximum magnitude 4 in a direction parallel to  $x$ -axis, find the values of  $a, b, c$ . [4]

*Or*

4. (a) Establish any one of the following : [4]

$$(i) \quad \nabla^2 \frac{(\bar{a} \cdot \bar{b})}{r} = 0$$

$$(ii) \quad \nabla \left( \frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})}{r^{n+2}} \bar{r}.$$

(b) For a solenoidal vector field  $\bar{E}$ , show that : [4]

$$\text{curl curl curl curl } \bar{E} = \nabla^4 \bar{E}$$

(c) Obtain  $f(k)$ , given that : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0; \quad k \geq 0, \quad F(0) = 0, \quad F(1) = 3$$

5. (a) Find the work done in moving a particle from the point  $A(0, 1, \frac{\pi}{4})$

to point  $B(\pi, 2, \frac{\pi}{2})$  in the force field : [4]

$$\bar{F} = (y \sin z - \sin x) \hat{i} + (x \sin z + 2yz) \hat{j} + (xy \cos z + y^2) \hat{k}.$$

(b) Evaluate : [5]

$$\iint_S (x^2 y^3 \hat{i} + z^2 x^3 \hat{j} + x^2 y^3 \hat{k}) \cdot d\bar{s},$$

where 's' is the surface of the sphere  $x^2 + y^2 + z^2 = a^2$ .

(c) Evaluate :

$$\iint_S (\nabla \times \bar{F}) \cdot d\bar{s},$$

where  $\bar{F} = (2y + x) \hat{i} + (x - y) \hat{j} + (z - x) \hat{k}$ ,

and 's' is the surface bounded by  $x = 0, y = 0, x + y + z = 1$ , which is not included in XOY-plane. [4]

*Or*

6. (a) Evaluate :

$$\int_c \bar{F} \cdot d\bar{r},$$

using Green's theorem, where :

$$\bar{F} = (2x^2 - y^2) \hat{i} + (x^2 + y^2) \hat{j},$$

and 'c' is the circle  $x^2 + y^2 = 1$  above x-axis. [4]

(b) Using Gauss's Divergence Theorem, evaluate :

$$\iint_S \bar{F} \cdot \hat{n} \, ds$$

where  $\bar{F} = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}$

and 's' is the surface of the sphere  $x^2 + y^2 + z^2 = 25$ . [5]

(c) Using Stokes' theorem, evaluate :

$$\int_C y dx + z dy + x dz,$$

where  $C$  is the curve of intersection of the surface  $x^2 + y^2 + z^2 = a^2$

by the plane  $x + z = a$ . [4]

7. (a) Show that analytic function  $f(z)$  with constant modulus is constant. [4]

(b) Evaluate :

$$\oint_C \frac{e^{3z}}{(z - \log 2)^4} dz$$

where  $C$  is the square with vertices  $\pm 1, \pm i$ . [5]

(c) Find the bilinear transformation, which sends the points  $1, i, -1$  from  $z$ -plane into points  $i, 0, -i$  of the  $w$ -plane. [4]

*Or*

8. (a) Show that the function :

$$u = x^4 - 6x^2y^2 + y^4$$

is harmonic and find the analytic function :

$$f(z) = u + iv. \quad [4]$$

(b) Evaluate :

$$\oint_c \frac{5z-2}{z(z-1)} dz$$

where  $c$  is  $|z| = 3$ . [5]

(c) Show that transformation :

$$w = z + \frac{1}{z} - 2i$$

maps the circle  $|z| = 2$  into an ellipse. [4]