May - June - 2011
SE - Elect. sem - II
2008 pattern

Total No. of Questions—12] [Total No. of Printed Pages—8+2

[3962]-210

S.E. (Comp./IT/Electrical/Instrumentation)

(II Sem.) EXAMINATION, 2011

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) In Section I attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

(*ii*) In Section II attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iii*) Answers to the two Sections should be written in separate answer-books.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

(*vi*) Use of electronic pocket calculator is allowed (Non-Programmable).

(*vii*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $\left(D^2 + 5D + 6\right) y = e^{-2x} \sec^2 x \,(1 + 2\tan x)$

(*ii*) $(3x + 2)^2 \cdot \dfrac{d^2y}{dx^2} + 3(3x + 2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 1$

P.T.O.

(iii) $\left(D^2 - 4D + 4\right) y = e^x \cos^2 x$

(iv) $\left(D^2 - 2D + 2\right) y = e^x \tan x$ [By Variation of Parameters Method].

(b) An emf $E \sin pt$ is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series. The current x satisfies the equation,

$$L \frac{dx}{dt} + \frac{1}{C} \int x \, dt = E \sin pt,$$

where

$$x = -\frac{dq}{dt}.$$

If

$$p^2 = \frac{1}{LC}$$

and initially the current x and charge q are zero, show that the current in the circuit at time t is given by

$$\frac{E}{2L} t \sin pt.$$ [5]

Or

2. (a) Solve the following (any *three*) : [12]

(i) $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$

(ii) $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = e^{e^x}$ [By Variation of Parameters Method]

(iii) $\frac{x^2 dx}{y^3} = \frac{y^2 dx}{x^3} = \frac{dz}{z}$

(iv) $\left(D^2 + D + 1\right) y = x \sin x.$

(b) Solve :

$$(D - 1)x + Dy = t$$

$$3x + (D + 4)y = t^2.$$ [5]

3. (a) Evaluate :

$$\int_{2+4i}^{5-5i} (x + iy + 1)\, dz$$

along the path $x = t^2 + 1$, $y = 3t + 1$. [5]

(b) If

$$v = -\frac{y}{x^2 + y^2},$$

find u such that $f(z) = u + iv$ is analytic and hence determine $f(z)$ in terms of z. [6]

(c) Find the map of the straight line $y = x$ under the transformation :

$$w = \frac{z - 1}{z + 1}.$$ [5]

Or

4. (*a*) Evaluate using residue theorem,

$$\oint_C \frac{2z^2 + 2z + 1}{(z+1)^3 (z-3)} dz$$

where C is the contour $|z + 1| = 2$. [6]

(*b*) Find the conditions under which,

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic. [5]

(*c*) Find the bilinear transformation, which maps the points 0, -1, i of z-plane on to the points 2, ∞, $\frac{1}{2}(5 + i)$ of the w-plane. [5]

5. (*a*) Using Fourier Integral representation prove that :

$$\int_0^\infty \frac{\cos\frac{\pi\lambda}{2} \cos \lambda x}{1 - \lambda^2} d\lambda = \frac{\pi}{2} \cos x \,, \quad |x| \le \frac{\pi}{2}$$

$$= 0 \quad , \quad |x| > \frac{\pi}{2}.$$ [5]

(*b*) Find the Fourier Sine and Cosine transforms of the following function :

$$f(x) = x \quad , \ 0 \le x \le 1$$

$$= 2 - x \ , \ 1 \le x \le 2$$

$$= 0 \quad , \ x > 2.$$ [6]

(c) Find z-transform of the following (any *two*) : [6]

(i) $f(k) = \dfrac{2^k}{k!}, \ k \geq 0$

(ii) $f(k) = e^{-3k} \cos 4k, \ k \geq 0$

(iii) $f(k) = (k+1)a^k, \ k \geq 0.$

Or

6. (a) Find inverse z-transform (any *two*) : [6]

(i) $F(z) = \dfrac{1}{(z-3)(z-2)}, \ 2 < |z| < 3$

(ii) $F(z) = \dfrac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{5}\right)}$ if $|z| < \dfrac{1}{5}$

(iii) $F(z) = \dfrac{10z}{(z-1)(z-2)}$ by integral inversion method.

(b) Solve the following :

$$f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k, \ k \geq 0, \ f(0) = 0.$$ [6]

(c) Solve the integral equation :

$$\int_0^\infty f(x) \sin \lambda x \, dx = 1 - \lambda, \ 0 \leq \lambda \leq 1$$

$$= 0, \qquad \lambda \geq 1.$$ [5]

SECTION II

7. (*a*) The first four moments about the working mean 44.5 of a distribution are –0.4, 2.99, –0.08 and 27.63. Calculate the moments about the mean. Also calculate the coefficients of skewness and kurtosis. [8]

(*b*) Calculate the regression equations of X on Y and Y on X from the following data and estimate X when Y = 26. Also find the coefficient of correlation : [9]

X	Y
10	5
12	6
13	7
17	9
18	13
20	15
24	20
30	21

Or

8. (*a*) Three groups of workers contain 3 men and 1 woman, 2 men and 2 women and 1 man and 3 women respectively. One worker is selected at random from each group. What is the probability that the group selected consists of 1 man and 2 women ? [5]

(*b*) The following table gives the number of days in a 50 day period during which automobile accidents occurred in a certain part of a city. Fit a Poisson's distribution to the data. Also calculate theoretical frequencies : [6]

No. of Accidents	**No. of Days**
0	19
1	18
2	8
3	4
4	1

(*c*) In a certain examination the percentage of passes and distinctions were 46 and 9 respectively. Assuming the distribution of marks to be normal, find the average marks obtained by the candidates if the minimum pass marks are 40 and distinction marks are 75 respectively.

(Given that corresponding to 0.04, standard normal variate is 0.1 and corresponding to 0.41, standard normal variate is 1.34.) [6]

9. (*a*) Find the constants a and b such that the surfaces $ax^2 - 2byz = (a + 4)x$ and $4x^2y + z^3 = 4$, are orthogonal at the point $(1, -1, 2)$. [5]

(*b*) If $\bar{a}, \bar{b}$ are constant vectors and $\bar{r}$ and r have their usual meaning, then show that (any *two*) : [6]

(*i*) $\nabla \times \left(\dfrac{\bar{a} \times \bar{r}}{r^3}\right) = \dfrac{3}{r^5}(\bar{a} \cdot \bar{r})\,\bar{r} - \dfrac{\bar{a}}{r^3}$

(*ii*) $\nabla \left(\dfrac{\bar{a} \cdot \bar{r}}{r^n}\right) = \dfrac{\bar{a}}{r^n} - \dfrac{n}{r^{n+2}}(\bar{a} \cdot \bar{r})\,\bar{r}$

(*iii*) curl $\left[(\bar{r} \times \bar{a}) \times \bar{b}\right] = \bar{b} \times \bar{a}$.

(*c*) If

$$\bar{r} \times \frac{d\bar{r}}{dt} = 0,$$

show that $\bar{r}$ has constant direction. [5]

Or

10. (*a*) Find the directional derivative of $\phi = x^2y^2 + y^2z^2 + z^2x^2$ at (1, 1, –2) in the direction of tangent to curve $x = e^{-t}$, $y = 2\sin t - 1$, $z = t - \cos t$ at $t = 0$. [5]

(*b*) Show that :

$$\bar{\mathrm{F}} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2},$$

is irrotational and that

$$\int_{\mathrm{C}} \bar{\mathrm{F}} \cdot d\bar{r} = 2\pi,$$

where C is any closed curve that contains the origin. [5]

(c) If $\bar{u}$ is a differentiable vector, then show that : [6]

(i) $\nabla(\bar{r} \cdot \bar{u}) = \bar{r} \times (\nabla \times \bar{u}) + (\bar{r} \cdot \nabla)\bar{u} + \bar{u}$ and

(ii) $\nabla \times (\bar{r} \times \bar{u}) = (\nabla \cdot \bar{u})\bar{r} - (\bar{r} \cdot \nabla)\bar{u} - 2\bar{u}$.

11. (a) Evaluate :

$$\int_C \bar{F} \cdot d\bar{r},$$

where

$$\bar{F} = \sin y\,\hat{i} + x(1 + \cos y)\,\hat{j}$$

and C is the boundary of an ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad z = 0.$$ [5]

(b) Evaluate :

$$\iint_S \nabla \times \bar{F} \cdot d\bar{s},$$

where

$$\bar{F} = (x^3 - y^3)\,\hat{i} - xyz\,\hat{j} + y^3\,\hat{k},$$

and S is the surface of $x^2 + 4y^2 + z^2 - 2z = 4$, above the plane $z = 0$. [5]

(c) Verify Stokes' theorem for

$$\bar{F} = y\,\hat{i} + (x - 2xz)\,\hat{j} - xy\,\hat{k}$$

and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$, above xoy-plane. [7]

Or

12. (*a*) Evaluate :

$$\int_C ydx + zdy + xdz,$$

where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [6]

(*b*) Evaluate :

$$\iint_S \left(4x\hat{i} + 2y\hat{j} - 3z\hat{k}\right) . d\bar{s}$$

over the area of ΔABC where A, B, C are the points where the plane $x + 2y + 2z = 6$ meet X, Y, Z axes respectively. [6]

(*c*) Show that :

$$\iint_S r\left(\nabla \frac{1}{r^3}\right) . d\bar{s} = \iiint_V \frac{3}{r^4} dV.$$ [5]