Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5668]-142

S.E. (Elect/E.&TC) (Second Semester) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt question Nos. **1** *or* **2**, **3** *or* **4**, **5** *or* **6**, **7** *or* **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^3 + 6D^2 + 12D + 8)y = e^{-2x} + 3^x + \cos 2x$

(*ii*) $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + \sin(\log x)$

(*iii*) $\frac{d^2y}{dx^2} + 4y = \tan 2x$ (By method of variation of parameter).

(*b*) Find Fourier sine transform of : [4]

$$f(x) = \begin{cases} \frac{\pi}{2} & 0 < x < \pi \\ 0 & x > \pi \end{cases}$$

P.T.O.

Or

2. (*a*) A 0.1 henry inductor, a 0.004 farad capacitor and a generator having e.m.f. given by 180 cos 40t, $t \geq 0$ are connected in series. Find the instantaneous charge Q and current I, if I = Q = 0 at t = 0. [4]

(*b*) Solve any *one* : [4]

(*i*) Find Z-transform of $\frac{2^k}{k}, k \geq 1$.

(*ii*) Find inverse z-transform of $\frac{1}{(z-3)(z-2)}$ for $2 < |z| < 3$.

(*c*) Find $f(k)$, given that : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0, f(0) = 0, f(1) = 3.$$

3. (*a*) Given : [4]

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}, \ y(1) = 1.$$

Evaluate $y(1.1)$ by Euler's modified method (Take $h = 0.1$).

(*b*) Find Lagrange's interpolating polynomial passing through set of points : [4]

x	0	1	2
y	2	3	6

Use it to find y at $x = 1.5$ and $\int_0^2 y \, dx$.

(*c*) Find directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) along the vector $\overline{i} + 2\overline{j} + 2\overline{k}$. [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla^2[\nabla.(\overline{r}/r^2)] = 2/r^4$

(*ii*) $\nabla\left(\frac{\overline{a}\,.\,\overline{r}}{r^2}\right) = \frac{\overline{a}}{r^2} - \frac{2(\overline{a}\,.\,\overline{r})}{r^4}\overline{r}$

(*b*) Show that the vector field $\overline{F} = (y^2 \cos x + z^2)\,\overline{i} + (2y \sin x)\overline{j} + 2xz\overline{k}$ is irrotational and find scalar potential ϕ such that $\overline{F} = \nabla\phi$. [4]

(*c*) Evaluate $\int_0^3 \frac{dx}{1+x}$ by dividing the interval into six parts using Simpson's $\frac{3}{8}$th rule correct upto four decimal places. [4]

5. (*a*) Evaluate the integral $\int_c \overline{F}\,.\,d\overline{r}$, where $\overline{F} = (3x^2 + 6y)i - 14yzj + 20\,xz^2k$ and C is the curve $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. [4]

(*b*) Verify Green's lemma for $\overline{F} = x^2 i + xyj$ over the region bounded by the boundaries : [5]

$$x = 0, y = 0, x = 1, y = 1$$

(*c*) Use Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = xy^2 i + yj + xz^2 k$ over the surface bounded by $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$. [4]

Or

6. (*a*) Use Green's Lemma to evaluate $\int_C (3y\,dx + 2x\,dy)$ where C is the boundary of the region bounded by $y = 0$, $y = \sin x$ for $0 \leq x \leq \pi$. [4]

(*b*) Find the work done by force field $\bar{F} = 2xy^2 i + (2x^2 y + y) j$ in taking a particle from (0, 0, 0) to (2, 4, 0) along the parabola $y = x^2$, $z = 0$. [4]

(*c*) Maxwell's equations are given by : [5]

$$\nabla \cdot \bar{E} = 0,\ \nabla \cdot \bar{H} = 0,\ \nabla \times \bar{H} = \frac{\partial \bar{E}}{\partial t},\ \nabla \times \bar{E} = -\frac{\partial \bar{H}}{\partial t}$$

then show that $\bar{E}$ satisfies the equation $\nabla^2 u = \dfrac{\partial^2 u}{\partial t^2}$.

7. (*a*) If $f(s) = u + iv$ is analytic function with constant amplitude, show that $f(z)$ is constant. [4]

(*b*) Evaluate :

$$\oint_C \cot z \, dz$$

where '*c*' is the circle $|z| = 4$. [5]

(*c*) Find the bilinear transformation, which maps the points $1, i, -1$ from z-plane onto the points $i, 0, -i$ of the w-plane respectively. [4]

Or

8. (*a*) If :

$$u = \frac{1}{2}\log(x^2 + y^2)$$

find v such that $f(z) = u + iv$ is analytic. [4]

(*b*) Evaluate : [4]

$$\oint_C \frac{z^2+1}{z-2}\,dz$$

where :

(*i*) C is the circle $|z - 2| = 1$,

(*ii*) C is circle $|z| = 1$.

(*c*) Show that under the transformation $w = z + \frac{1}{z}$ family of circles $r = c$ are mapped onto family of ellipses. What happens if $r = 1$. [5]