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[5057]-2031

S.E. (Electrical and Instru.) (First Semester)

EXAMINATION, 2016

ENGINEERING MATHS

Paper III

(2015 PATTERN)

Time : Two Hours

Maximum Marks : 50

N.B. :— (i) Figures to the right indicate full marks.

(ii) Use of electronic pocket calculator is allowed.

(iii) Assume suitable data, if necessary.

(iv) Neat diagrams must be drawn wherever necessary.

1. (a) Solve any two : [8]

(i) $(D^2 - 4D + 3)y = x^3 e^{2x}$

(ii) $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 2 \sin [\log (1+x)]$

(iii) $(D^2 - 1)y = \frac{2}{1 + e^x}$ using method of variation of parameters.

(b) Solve using Laplace transforms : [4]

$$\frac{d^2 y}{dx^2} + y = t,$$

given $y(0) = 1, \quad y'(0) = -2$.

P.T.O.

Or

2. (a) A circuit consists of an inductance L and condenser of capacity C in series. An e.m.f. $\Sigma \sin nt$ is applied to it at time $t = 0$, the initial charge and initial current being zero, find the current flowing in the circuit at any time

$$t \text{ for } \frac{1}{\sqrt{LC}} \neq n. \quad [4]$$

- (b) Solve any one : [4]

- (i) Find :

$$L \left[\frac{e^{-at} - e^{-bt}}{t} \right].$$

- (ii) Find :

$$L^{-1} \left[\frac{s+7}{s^2 + 2s + 2} \right].$$

- (c) Find Laplace transform of : [4]

$$L [\sin t \, U(t-4)].$$

3. (a) Solve the integral equation : [4]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}$$

where $\lambda > 0$.

(b) Solve any one : [4]

(i) Find z -transform of $f(k) = \frac{2^k}{k}, k \geq 1$.

(ii) Find inverse z -transform of :

$$F(z) = \frac{1}{(z-a)^2}, |z| < a.$$

(c) If directional derivative of : [4]

$$\phi = ax^2y + by^2z + cz^2x$$

at (1, 1, 1) has maximum magnitude 15 in the direction parallel

to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, hence find the values of a, b, c .

Or

4. (a) Attempt any one : [4]

(i) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(ii) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r).$

(b) Find the values of the constant scalars a, b, c if the vector point function :

$$\bar{V} = (x + 2y + az)i + (bx - 3y + z)j + (4x + cy + 2z)k$$

is irrotational. [4]

(c) Obtain $f(k)$, given that : [4]

$$f_{k+2} - 4f_k = 0, k \geq 0, f(0) = 0, f(1) = 2.$$

5. Attempt any two :

- (a) Using Green's theorem, show that the area bounded by a simple closed curve C is given by :

$$\frac{1}{2} \int (x dy - y dx).$$

Hence find the area of the ellipse $x = a \cos \theta$, $y = b \sin \theta$. [6]

- (b) Use the divergence theorem to evaluate : [6]

$$\iiint_s (y^2 z^2 i + z^2 x^2 j + x^2 y^2 k) \cdot \overline{dS}$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 9$ above the xoy plane.

- (c) Verify Stokes' theorem for : [7]

$$\overline{F} = (y - z + 2)i + (yz + 4)j + xz k$$

over the surface $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$.

Or

6. Attempt any two :

- (a) Evaluate $\int_c \overline{f} \cdot d\overline{r}$ where

$$\overline{f} = (5xy - 6x^2)i + (2y - 4x)j$$

and c is the arc of the curve in the xoy plane, $y = x^3$ from $(1, 1)$ to $(2, 8)$ [6]

(b) Evaluate $\iint_s \bar{F} \cdot \overline{ds}$ where

$$\bar{F} = yzi + zxj + xyk$$

and s is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. [6]

(c) Use Stokes' theorem to evaluate : [7]

$$\int_c (4yi + 2zj + 6yk) \cdot d\bar{r}$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = 2z$ and $x = z - 1$.

7. (a) If

$$v = \frac{-y}{x^2 + y^2},$$

find u such that, $u + iv$ is analytic function. [4]

(b) Evaluate :

$$\oint_c \frac{z+4}{z^2+2z+5} dz,$$

where c is a circle $|z-2i| = 3/2$. [5]

(c) Find the bilinear transformation which maps points $0, -1, \infty$ of z -plane onto $-1, -(2+i), i$ of W -plane. [4]

Or

8. (a) Find the condition satisfied by a , b , c and d under which,

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic function. [4]

- (b) Evaluate :

$$\int_0^{2\pi} \frac{d\theta}{5 - 3 \cos \theta}$$

using Cauchy's theorem. [5]

- (c) Find the image of st. line $y = x$ under the transformation : [4]

$$W = \frac{z-1}{z+1}.$$