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[5352]-541**S.E. (Electrical Engg./Instru. & Control) EXAMINATION, 2018****ENGINEERING MATHEMATICS—III****(2015 PATTERN)****Time : Two Hours****Maximum Marks : 50****N.B. :— (i) Figures to the right indicate full marks.****(ii) Use of electronic pocket calculator is allowed.****(iii) Net diagrams must be drawn wherever necessary.****(iv) Assume suitable data, if necessary.**1. (a) Solve any *two* : [8]

(1) $\frac{d^3y}{dx^3} + 4 \frac{dy}{dx} = \sin 2x$

(2) $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$ by variation of parameters method

(3) $(4x + 1)^2 \frac{d^2y}{dx^2} + 2(4x + 1) \frac{dy}{dx} + y = 2x + 1$

(b) Solve by Laplace Transform method [4]

$$\frac{d^2y}{dt^2} + 9y(t) = 18t$$

with $y(0) = 0, y(\pi/2) = 0$.

P.T.O.

Or

2. (a) An inductor of 0.5 henry is connected in series with resistor of 6 ohms. A capacitor of 0.02 farad and generator having alternative voltage given by $24 \sin 10t$ ($t > 0$) with a switch K. Forming a differential equation find the current and charge at any time t if charge is zero when switch is closed at $t = 0$. [4]

- (b) Solve any one : [4]

$$(1) \quad L \left[t \int_0^t e^{-4t} \sin 3t \, dt \right]$$

$$(2) \quad L^{-1} \left[\frac{2s+1}{(s^2 + s + 1)^2} \right]$$

- (c) Find Laplace transform of $(1 + 2t - 3t^2 + 4t^3) \cup (t - 2)$. [4]

3. (a) Find Fourier sine transform of $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$ [4]
- (b) Attempt any one : [4]

- (i) Find z-transform of $f(k) = (k + 1)(k + 2)2^k$ $k \geq 0$.

- (ii) Show that

$$z^{-1} \left\{ \frac{1}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)} \right\} = \{x_k\} \text{ for } |z| > \frac{1}{2}$$

$$\text{where } x_k = 6 \left[\left(\frac{1}{2}\right)^{k-1} - \left(\frac{1}{3}\right)^{k-1} \right], \quad k \geq 1.$$

- (c) Find directional derivative of $\phi = xy^2 + yz^3$ at $(2, -1, 1)$ along the line $2(x - 2) = (y + 1) = (z - 1)$. [4]

Or

4. (a) Prove any one : [4]

$$(i) \quad \bar{a} \cdot \nabla \left[\bar{b} \cdot \nabla \left(\frac{1}{r} \right) \right] = \frac{3(\bar{a} \cdot \bar{r})(\bar{b} \cdot \bar{r})}{r^5} - \frac{(\bar{a} \cdot \bar{b})}{r^3}$$

$$(ii) \quad \nabla \cdot 4e^r = e^r + \frac{4}{r}e^r$$

- (b) Show that $\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$ is irrotational and find ϕ such that $\bar{F} = \nabla\phi$. [4]

$$(c) \quad \text{Solve } y_k - \frac{5}{6}y_{k-1} + \frac{1}{6}y_{k-2} = \left(\frac{1}{2}\right)^k \quad k \geq 0. \quad [4]$$

5. Attempt any two :

- (a) Evaluate $\int_c \bar{F} \cdot d\bar{r}$ for $\bar{F} = (2x + y)\bar{i} + (3y - x)\bar{j}$ and c is the straight line joining $(0, 0)$ and $(3, 2)$. [6]

- (b) Apply Stokes' theorem to evaluate

$$\int_c 4y \, dx + 2z \, dy + 6y \, dz$$

where c is the curve of intersection of $x^2 + y^2 + z^2 = 6z$ and $z = x + 3$. [7]

- (c) Evaluate $\iint_s \bar{r} \cdot \hat{n} \, d\bar{s}$ over the surface of a sphere of radius 1 with centre at the origin. [6]

Or

6. Attempt any two :

- (a) Using Green's theorem evaluate $\int_c \bar{F} \cdot d\bar{r}$ where

$$\bar{F} = (2x - \cos y)\bar{i} + x(4 + \sin y)\bar{j}$$

where c is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 0$. [6]

(b) Evaluate $\iint_s (\nabla \times \bar{\mathbf{F}}) \cdot d\bar{s}$ for $\bar{\mathbf{F}} = y\bar{i} + z\bar{j} + x\bar{k}$ where s is the surface of paraboloid $z = 1 - x^2 - y^2$ above the XOY plane. [7]

(c) Use Gauss divergence theorem to evaluate $\iint_s \bar{\mathbf{F}} \cdot d\bar{s}$ over the cylindrical region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = \alpha$, where $\bar{\mathbf{F}} = x\bar{i} + y\bar{j} + z^2\bar{k}$. [6]

7. (a) If $V = \sinh x \cos y$ find u such that $u + iv$ is analytic function. [4]

(b) Evaluate $\oint_c \frac{1+z}{z(z-2)} dz$ where c is the circle $|z| = 1$. [4]

(c) Find the bilinear transformation which maps points $1, i, -1$ of z -plane onto $i, o, -i$ of w -plane. [5]

Or

8. (a) Find ' a ' such that the function $f(z) = r^2 \cos 2\theta + ir^2 \sin(a\theta)$ is an analytic function. [4]

(b) Evaluate $\oint_c \frac{15z+9}{z(z+3)} dz$ where c is the circle $|z - 1| = 3$. [4]

(c) Show that under the transformation $w = \frac{i-z}{i+z}$, x -axis in z -plane is mapped onto the circle $|w| = 1$. [5]