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[4957]-210**S.E. (Electrical/Inst./Comp./I.T.) (I Sem.) EXAMINATION, 2016****ENGINEERING MATHEMATICS—III****(2008 PATTERN)****Time : Three Hours****Maximum Marks : 100**

N.B. :— (i) In Section I, solve Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6. In Section II, solve Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(ii) Answers to the two Sections should be written in separate answer-books.

(iii) Figures to the right indicate full marks.

(iv) Assume suitable data, if necessary.

(v) Neat diagrams must be drawn wherever necessary.

(vi) Use of non-programmable electronic pocket calculator is allowed.

SECTION I

1. (a) Solve any *three* of the following : [12]

(i) $(D^2 - 4D + 3)y = x^3 e^{2x}$

(ii) $(D^2 + D + 1)y = x \sin x$

P.T.O.

(iii) $(D^2 - 2D + 2)y = e^x \tan x$ (by using method of variation of parameters)

(iv) $(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin [\log(1 + x)]$

(b) Solve : [5]

$$\frac{dx}{dt} + 2x - 3y = t,$$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}.$$

Or

2. (a) Solve any *three* of the following : [12]

(i) $(D^2 + 2D + 1)y = xe^{-x} \cos x$

(ii) $(D^2 + 1)y = x^2 \sin 2x$

(iii) $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$ (by using method of variation of parameters)

(iv) $\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x}.$

(b) An electric circuit consists of an inductance 0.1 henry, a resistance R of 20Ω and a condenser of a capacitance 25 microfarads. If the differential equation of electric circuit is :

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{c} = 0$$

then find the charge q and current i at any time t , given

that at $t = 0$, $q = 0.05$ coulombs, $i = \frac{dq}{dt} = 0$ when $t = 0$. [5]

3. (a) If $v = 3x^2y - y^3$, find its harmonic conjugate u . Find $f(z) = u + iv$ in terms of z . [5]
- (b) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find the centre of the ellipse and its semi-major and minor axes. [5]
- (c) Evaluate :

$$\oint_C \frac{4z^2 + z}{(z-1)^2} dz$$

where 'C' is the contour $|z - 1| = 2$. [6]

Or

4. (a) Determine k such that the function :

$$f(z) = e^x \cos y + ie^x \sin ky$$

is analytic. [5]

- (b) Find the bilinear transformation, which maps the points $z = -1, 0, 1$ onto the points $w = 0, i, 3i$. [5]
- (c) Evaluate :

$$\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4 \cos \theta} d\theta$$

using Cauchy's theorem. [6]

5. (a) Find the Fourier transform of : [6]

$$f(x) = \begin{cases} 1 - x^2 & , \quad |x| \leq 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

Hence show that :

$$\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx = \frac{-3\pi}{16}.$$

(b) Find the Fourier cosine transform of the function : [5]

$$f(x) = \begin{cases} \cos x & , \quad 0 < x < a \\ 0 & , \quad x > a \end{cases}$$

(c) Find z -transform of (any two) : [6]

(i) $f(k) = 4^k \cdot \sin(2k + 3), k \geq 0$

(ii) $f(k) = k \cdot 5^k, k \geq 0$

(iii) $f(k) = 4^k + 5^k, k \geq 0.$

Or

6. (a) Find inverse z -transform of (any two) : [8]

(i) $F(z) = \frac{1}{(z-2)(z-3)}, 2 < |z| < 3$

(ii) $F(z) = \frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}, |z| > \frac{1}{2}$

(iii) $F(z) = \frac{z(z+1)}{z^2 - 2z + 1}, |z| > 1.$

(b) Solve the difference equation : [4]

$$12f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0, f(0) = 0,$$

$$f(1) = 3.$$

- (c) By considering Fourier sine integrals of e^{-mx} ($m > 0$) prove that : [5]

$$\int_0^{\infty} \frac{\lambda \sin \lambda x}{\lambda^2 + m^2} d\lambda = \frac{\pi}{2} e^{-mx}, \quad m > 0, x > 0.$$

SECTION II

7. (a) Compute the first four moments about the mean, coefficient β_1 and β_2 for the following frequencies distribution : [8]

Number of Jobs Completed	Number of Workers
0—10	6
10—20	26
20—30	47
30—40	15
40—50	6

- (b) The equation of two lines of regression are given by :

$$2x = 8 - 3y \text{ and}$$

$$2y = 5 - x.$$

Find the mean values of x and y . Also obtain the value of correlation coefficient. [9]

Or

8. (a) Two cards are drawn from a well shuffled pack of 52 cards.

Find the probability that they are both kings if :

(i) The first card drawn is replaced

(ii) The first card is not replaced. [6]

- (b) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped find the probability that exactly two will strike the target. [6]

- (c) In a sample of 1000 cases, the mean of certain test is 14 and S.D. is 25. Assuming the distribution is normal, find how many scores above 18, given, $A(z = 0.16) = 0.0160$. [5]

9. (a) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at $(1, -1, 1)$ along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [5]

- (b) Show that :

$$\bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$$

is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$. [6]

- (c) Show that : [5]

$$\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^2} \right) \right] = \frac{2}{r^4}.$$

Or

10. (a) For the curve :

$$x = e^t \cos t,$$

$$y = e^t \sin t,$$

$$z = e^t$$

find the velocity and acceleration of the particle moving on the curve at $t = 0$. [5]

(b) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find : [6]

(i) $\text{div}(\text{grad } \phi)$

(ii) $\text{curl}(\text{grad } \phi)$.

(c) Find angle between normal to surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point $(1, 1, 1)$. [5]

11. (a) Evaluate :

$$\int_C \vec{F} \cdot d\vec{r} \quad \text{for } \vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$$

along the path of the straight line joining $(0, 0, 0)$ and $(2, 1, 3)$. [5]

(b) Using Green's theorem, find :

$$\oint u dx + v dy$$

for the vector field :

$$\vec{F} = x^2\vec{i} + xy\vec{j}$$

over the region R enclosed by $y = x^2$ and then line $y = x$. [6]

- (c) Using divergence theorem, evaluate :

$$\iint_S \vec{F} \cdot \hat{n} \, dS, \text{ for } \vec{F} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$$

where S : surface bounded by coordinate planes at the plane
 $2x + y + 2z = 6$. [6]

Or

12. (a) Evaluate :

$$\iint \vec{r} \cdot \hat{n} \, dS$$

where S is a closed surface. [5]

- (b) Find the work done in moving a particle once round the circle
 $x^2 + y^2 = a^2; z = 0$ under the field of force : [6]

$$\vec{F} = \sin y \vec{i} + x(1 + \cos y) \vec{j}.$$

- (c) Evaluate :

$$\iint (\nabla \times \vec{F}) \cdot \hat{n} \, dS$$

where

$$\vec{F} = (x + y)\vec{i} + (x^2 + yz)\vec{j} - 3xy^2\vec{k}$$

and S is surface of cone $z = 4 - \sqrt{x^2 + y^2}$ above X o Y
 plane. [6]