Total No of Questions: [12] ***SEAT NO. :***

[Total No. of Pages : 3]

S.E. (COMP / IT / ELECTRICAL / INSTRUMENTATION)
ENGINEERING MATHEMATICS III
2008 Course

Time: 3 Hours ***Max. Marks : 100***

Instructions to the candidates:

1) ***In Section I attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.***
2) ***In Section II attempt Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.***
3) ***Answers to the two sections should be written in separate answer books.***
4) ***Neat diagrams must be drawn wherever necessary.***
5) ***Figures to the right side indicate full marks.***
6) ***Use of Calculator(non programmable) is allowed.***
7) ***Assume Suitable data if necessary***

SECTION I

Q1) a) Solve the following (any three) [12]

i) $(D^2 + 4)y = \sin x \cos 3x$

ii) $(2x+1)^2 \frac{d2y}{dx^2} - 2(2x+1)\frac{d2y}{dx^2} - 12y = 6x$

iii) $(D^2 + 9)y = \tan 3x$ (by variation of parameters)

iv) $(D^2 - 3D + 2)y = e^{e^{-x}}$

b) A circuit consists of an inductance L & condenser of capacity C in series. An alternating emf Esin(nt) is applied to it. At time t = 0, the initial current and charge on the condenser being zero, find the current flowing in the circuit at any time t when $\omega \neq n$, if $\omega^2 = 1 / LC$ [5]

OR

Q2) a) Solve the following (any three) [12]

i) $x^2 \frac{d2y}{dx^2} - x\frac{dy}{dx} + y = x\log x$

ii) $(D^2 - 4D + 4)y = \frac{e^{2x}}{x}$

iii) $(D^2 + 2D + 1)y = e^{-x} \log x$ (by variation of parameters)

iv) $\frac{x^2 dx}{y^3} = \frac{y^2 dy}{x^3} = \frac{dz}{z}$

b) Solve : $\frac{du}{dx} + v = \sin x,\ \frac{dv}{dx} + u = \cos x$ [5]

given at x = 0, u = 1 and v = 0

Q3) a) Evaluate $\int_0^{1+i}(z^2+1)dz$ along the straight line. [5]

b) If $u = \frac{1}{2}\log(x^2 + y^2)$, find v such that f(z) = u + iv is analytic. Determine f(z) in terms of z. [6]

c) Show that the transformation $\omega = (z-b)/(z+b)$ maps the right half of the z – plane into the unit circle $|\omega| < 1$. [5]

OR

Q4) a) Evaluate using residue theorem, $\oint_c \frac{z+3}{(z-2)(z+1)^2} dz$ where c is the boundary of the square with vertices (±1.5, ±1.5) [6]

b) If f(z) is analytic, [5]

show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\,|f'(z)|^2$

c) Find the bilinear transformation, which maps the point -1, 0, 1 of z – plane on to the points 0, i, 3i of the ω - plane [5]

Q5) a) Using Fourier Integral representation [5]

prove that $\int_0^\infty \frac{2\lambda \sin\lambda x}{\lambda^4 + 4}\, d\lambda = \frac{\pi}{2} e^{-x} \sin x$

b) Find the Fourier sine and cosine transforms of the following function: [6]

$$f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$$

c) Find the z transform of the following (**any two**) [6]

i. $f(k) = 3^k \cos(4k + 5),\ k \geq 0$

ii. $f(k) = \frac{5^k}{k},\ k \geq 1$

iii. $f(k) = k2^k,\ k \geq 0$

OR

Q6) a) Find inverse z – transform (**any two**) [6]

i. $F(z) = \frac{z}{(z-1)(z-2)},\ |z| \geq 2$

ii. $F(z) = \frac{z^2}{z^2+1}$ by integral inversion method

iii. $F(z) = \frac{3z^2 + 2z}{z^2+3z+2},\ 1 < |z| < 2$

b) Solve the following: [6]

$12f(k+2) - 7f(k+1) + f(k) = 0,\ k \geq 0,\ f(0)=0,\ f(1) = 3$

c) Solve the Integral equation [5]

$$\int_0^\infty f(x)\cos\lambda x\,dx = \begin{cases} 1-\lambda, & 0 \leq \lambda \leq 1 \\ 0, & \lambda \geq 1 \end{cases}$$

And hence show that

$$\int_0^{\infty} \frac{sin^2 z}{z^2} dz = \frac{\pi}{2}$$

SECTION II

Q7) a) Calculate the first four central moments of the given distribution. Is the distribution platykurtic? [9]

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
f	4	36	60	90	70	40	10

b) The regression equations are 8x – 10y + 66 = 0 and 40x – 18y - 21 = 0. The value of variance of x is 49. Find the correlation coefficient between x and y. Also find the standard deviation of y. [8]

OR

Q8) a) The average number of misprints per page of a book is 2. Assuming the distribution of number of misprints to be Poisson, find the probability that a particular book is free from misprints and containing more than one misprint. [6]

b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of distribution. [6]

(Given Area corresponding z = 0.5 is 0.19 and z = 1.41 is 0.42)

c) The mean and variance of Binomial distributions are 6 and 2 respectively. Find p(r ≥ 1). Where r is the number of successes in n trial. [5]

Q9) a) If the directional derivative of Ø = axy + byz + czx at (1,1,1) has maximum magnitude 8 in a direction parallel to z – axis, find the values of a, b and c. [6]

b) Show that $\bar{F} = (x^2 + xy^2)\bar{\imath} + (y^2 + x^2y)\bar{\jmath}$ is irrational and final scalar Ø such that $\bar{F} = \nabla$ Ø [6]

c) If $\bar{u}$ and $\bar{v}$ are irrotational vectors then, [4]

prove that $\bar{u}$ x $\bar{v}$ is solenoidal vector.

OR

10) a) Find the directional derivatives of Ø = $\sqrt{3}\, e^{x+y+z}$ at (0, 0, 0) along a line equally inclined with co – ordinate axes. [5]

b) Determine f(r) such that f(r)$\bar{r}$ is solenoidal. [3]

c) Show that **(any two)** [8]

i. $\nabla\left(\frac{\bar{a}.\bar{r}}{r^2}\right) = \frac{\bar{a}}{r^2} - \frac{2\overline{(a.r)}}{r^4}\bar{r}$

ii. $\nabla\, x\left[\bar{a}\, x\,\overline{(b}\, x\, \bar{r})\right] = \bar{a} \text{ x } \bar{b}$

iii. $For\ a\ scalar\ function\ u\ and\ v, show\ that$

$\nabla.(u\nabla\nabla - V\nabla u) = u\nabla^2V - V\nabla^2u$

11) a) Using Green's theorem, Evaluate $\oint_c \left(e^y\bar{\imath} + x(1 + e^y)\bar{\jmath}\right).d\bar{r}$ for a closed curve C given by $\frac{x^2}{36} + \frac{y^2}{49} = 1, z = 0$ [6]

b) Evaluate [6]

$\iint_s (\nabla \text{ x } \bar{F}).\hat{n}ds$ where 's' is the curved surface of the paraboloid $x^2 + y^2 = 4z$ bounded by the plane z = 4, where $\bar{F} = 3(x-y)\bar{\imath} + 2xz\bar{\jmath} + xy\bar{k}$.

c) Show that $\iiint_v \frac{2\,dv}{r} = \iint_s \frac{\bar{r}.\hat{n}}{r}ds$ [5]

OR

12) a) Evaluate $\int_c \bar{F}.d\bar{r}$ for $\bar{F} = 3x\bar{\imath} + (x-y)\bar{\jmath} + z\bar{k},$ [6]

$along\ the\ curve\ x = 3t, y = t, z = t^2\ from\ t = 0\ to\ t = 1.$

b) Apply stokes theorem to evaluate $\int_c 4ydx + 2zdy + 6ydz$, where c is the curve of intersection of $x^2 + y^2 + z^2 = 9$ and x +z = 0 [6]

c) If $\nabla.\bar{A} + \frac{1}{c}\frac{\partial\emptyset}{\partial t} = 0$ and $\nabla^2\bar{A} = \frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2}$ then [5]

Show that $\bar{E} = -\nabla\emptyset - \frac{1}{c}\frac{\partial\bar{A}}{\partial t}, \bar{H} = \nabla \text{ x } \bar{A}$

are solution of Maxwell's equation $\nabla \text{ x } \bar{H} = \frac{1}{c}\frac{\partial \bar{E}}{\partial t}$