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## S.E. (Comp/IT) (II Semester) EXAMINATION, 2019 <br> ENGINEERING MATHEMATICS-III

## (2015 PATTERN)

Time : Three Hours
Maximum Marks : 60
N.B. :- (i) Neat diagrams must be drawn wherever necessary.
(ii) Figures to the right indicate fulk marks.
(iii) Use of electronic pocket calculator is allowed.
(iv) Assume suitable data, if necessary.

1. (a) Solve any two differential equations :
(i) $\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d x}+4 y=e^{2 x} \sin 4 x+2^{3 x}+6$
(ii) $x^{2} \frac{d^{2} y}{d x^{2}}+x \frac{d y}{d x}-y=x^{4}+3 x+1$
(iii) $\frac{d^{2} y}{d x^{2}}+9 y=\tan 3 x$, by using the method of variation of parameters.
(b) Solve the integral equation :

$$
\int_{0}^{\infty} f(x) \cos \lambda x d x=\left\{\begin{array}{cc}
2-\lambda, & 0 \leq \lambda \leq 2 \\
0, & \lambda>2
\end{array}\right.
$$

2. (a) An inductor of 0.25 henries, with negligible resistance, a capacitor of 0.04 farads are connected in series and having an alternating voltage [12 sin $6 t$ ]. Find the current and charge at any time $t$.
(b) Solve any one of the following :
(i) Obtain $z\left[4^{k} e^{-6 k}\right], k \geq 0$.
(ii) Obtain $z^{-1}\left[\frac{13 z}{(5 z+1)(4 z+1)}\right]$.
(c) Solve the difference equation :
[4]

$$
f(k+2)-7 f(k+1)+12 f(k)=0
$$

where $f(0)=0, f(1)=3,-k \geq 0$.
3. (a) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50 . Obtain the first four central moments, $\beta_{1}$ and $\beta_{2}$.
[4]
(b) Fit a straight line of the form $\mathrm{Y}=a \mathrm{X}+b$ to the following data by the least square method :

| X | 1 | 3 | 4 | 5 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y | -3 | 1 | 3 | 5 | 7 | 11 |

(c) A riddle is given to three students whose probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that the riddle is solved.
4. (a) In a sample of 1,000 cases, the mean of a certain examination is 14 and standard deviation is 2.5 . Assuming the distribution to be normal. Find the number of students scoring between 12 and 15.
[4]
[Given: $\mathrm{Z}_{1}=0.4, \mathrm{~A}_{1}=0.1554, \mathrm{Z}_{2}=0.8, \mathrm{~A}_{2}=0.2881$ ]
(b) During working hours, on an average 3 phone calls are coming into a company within an hour. Using Poisson distribution, find the probability that during a particular working hour, there will be at the most one phone call,
(c) For a bivariate data, the regression equation of Y on X is $8 x-10 y=-66$ and the regression equation of X on Y is $40 x-18 y=214$. Find the mean values of X and Y . Also, find the correlation coefficient between X and Y .
5. (a) Find the directional derivative of $\phi=x y^{2}+y z^{2}+z x^{2}$ at $(1,1,1)$ along the line $2(x-2)=y+1=z-1$.
(b) Find constants $a, b, c$ so that

$$
\overline{\mathrm{F}}=(x+2 y+a z) \bar{i}+(b x-3 y-z) \bar{j}+(4 x+c y+2 z) \bar{k}
$$

is irrotational.
[4]
(c) Find the workdone by the force

$$
\overline{\mathrm{F}}=\left(x^{2}-y z\right) \bar{i}+\left(y^{2}-z x\right) \bar{j}+\left(z^{2}-x y\right) \bar{k}
$$

in taking a particle from $(0,0,0)$ to $(1,2,1)$.
6. (a) Show that (any one) :
(i) $\nabla \cdot\left(\frac{\bar{a} \times \bar{r}}{r^{n}}\right)=0$
(ii) $\nabla^{2}\left[\nabla \cdot\left(\frac{\bar{r}}{r^{2}}\right)\right]=\frac{2}{r^{4}}$.
(b) Find the directional derivative of $\phi=4 x z^{3}-3 x^{2} y^{2}$ at ( $2,-1,2$ ) along the tangent to the curve

$$
\begin{equation*}
x=e^{t} \cos t, y=e^{t} \sin t, z=e^{t} \text { at } t=0 . \tag{4}
\end{equation*}
$$

(c) Find the workdone by, $\overline{\mathrm{F}}=2 x y^{2} \bar{i}+\left(2 x^{2} y+y\right) \vec{j}$ in taking a particle from ( $0,0,0$ ) to ( $2,4,0$ ) along the parabola $y=x^{2}, z=0$.
7. (a) Determine the analytic function $f(z)=u+i v$ if $u=$ $2 x-x^{3}+3 x y^{2}$.
(b) Find the bilinear transformation that maps to points $z=-i, 0, i$ into the points $\mathrm{W}=1,0, \infty$.
(c) Evaluate $\int_{c} \frac{z^{3}}{z^{2}-4} d z$ where $c$ is the circle $|z|=3$.

## Or

8. (a) Determine the analytic function $f(z)=\ddot{y}+i v$ if $u=3 x^{2} y+2 x^{2}-y^{3}-2 y^{2}$.
(b) Find image of the circle $|z-2 i|=2$, under the mapping $w=\frac{1}{2}$.
(c) Evaluate $\int_{c} \frac{2 z^{2}+z}{z^{2}-1} d z$, where $c$ is the circle $|z|=\frac{3}{2} \cdot[5]$
