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[5152]-166**S.E. (Comp./IT) (II Semester) EXAMINATION, 2017****ENGINEERING MATHEMATICS—III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50**

N.B. : (i) Attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

- (ii) Neat diagrams must be drawn wherever necessary.
- (iii) Figures to the right indicate full marks.
- (iv) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.
- (v) Assume suitable data, if necessary.

1. (a) Solve any two : [8]

$$(i) (D^2 + 1)y = x \cos x$$

$$(ii) (D^2 - 4D + 4)y = 8x^2 e^{2x} \sin x$$

$$(iii) (D^2 + 9)y = \frac{1}{1 + \sin 3x}.$$

(b) Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$. [4]

P.T.O.

Or

2. (a) An e.m.f. $E \sin pt$ is applied at $t = 0$ to a circuit containing a condenser C and inductance L in series the current I satisfies the equation : [4]

$$L \frac{dI}{dt} + \frac{1}{C} \int I dt = E \sin pt, \text{ where}$$

$$i = \frac{dq}{dt}, \text{ If } p^2 = \frac{1}{LC}$$

and initially the current and the charge are zero, find current at any time t .

- (b) Find the inverse z -transform (any one) : [4]

$$(i) F(z) = \frac{z}{(z-1)(z-2)}, |z| > 2$$

$$(ii) F(z) = \frac{1}{(z-2)(z-3)}, 2 < |z| < 3$$

- (c) Solve the following difference equation to find $f(k)$: [4]

$$6f(k+2) - 5f(k+1) + f(k) = 0$$

$$f(0) = 0, f(1) = 3, k \geq 0.$$

3. (a) The first four moments of a distribution about 25 are -1.1, 89, -110 and 23300. Calculate the first four moments about the mean. [4]

- (b) In a Poisson distribution if : [4]

$$P(r = 1) = 2 P(r = 2)$$

then show that :

$$P(r = 3) = 0.0613.$$

- (c) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ towards the point $\bar{i} + \bar{j} - \bar{k}$. [4]

Or

4. (a) Attempt any one : [4]

- (i) For scalar functions ϕ and ψ , show that :

$$\nabla \cdot (\phi \nabla \psi - \psi \nabla \phi) = \phi \nabla^2 \psi - \psi \nabla^2 \phi.$$

- (ii) Show that :

$$\nabla^2 \left(\frac{\bar{a} \cdot \bar{b}}{r} \right) = 0.$$

- (b) Show that the vector field :

$$\bar{F} = (ye^{xy} \cos z) \bar{i} + (xe^{xy} \cos z) \bar{j} + (-e^{-xy} \sin z) \bar{k}$$

is irrotational. Also find the corresponding scalar ϕ , such that

$$\bar{F} = \nabla \phi. \quad [4]$$

- (c) If the two lines of regression are $9x + y - \lambda = 0$ and $4x + y - \mu = 0$ and the means of x and y are 2 and -3 respectively, then find λ , μ and the coefficient of correlation between x and y . [4]

5. (a) Apply Green's Lemma to evaluate the : [5]

$$\oint (3x^2 - 8y^2) dx + (4y - 6xy) dy,$$

where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$ in the plane $z = 0$.

(b) If :

[4]

$$\bar{F} = (x^2 + y - 4)i + 3xy\hat{j} + (2xz + z^2)\hat{k},$$

evaluate :

$$\iint_S (\nabla \times \bar{F}) \cdot \hat{n} dS,$$

where S is the surface of the sphere :

$$x^2 + y^2 + z^2 = 16$$

above the xy plane.

(c) Evaluate :

[4]

$$\iint_S \bar{F} \cdot \hat{n} dS,$$

where

$$\bar{F} = (2x + 3y^2z^2)i - (x^2z^2 + y)\hat{j} + (y^3 + 2z)\hat{k}$$

and S is the surface of the sphere with centre (3, -1, 2) and radius 3.

Or

6. (a) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ from the point (0, 0, 0) to (1, 1, 1) along the curve $x = t, y = t^2, z = t^3$, given : [4]

$$\bar{F} = xy\hat{i} - z^2\hat{j} + xyz\hat{k}.$$

- (b) Using divergence theorem, evaluate $\iint_S \bar{F} \cdot \hat{n} dS$, over S, the surface of unit cube bounded by the co-ordinates planes and the planes $x = 1, y = 1$ and $z = 1$ where $\bar{F} = 2xi + 3y\hat{j} + 4z\hat{k}$. [4]

- (c) Apply Stokes' theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$, where $\bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$,
where C is the curve given by : [5]

$$x^2 + y^2 + z^2 - 2ax - 2ay = 0$$

and

$$x + y = 2a.$$

7. (a) If $v = 4xy(x^2 - y^2)$, find u such that $f(z) = u + iv$ is analytic and determine $f(z)$ in terms of z . [4]

- (b) Evaluate $\oint_C \tan z dz$ where C is the circle $|z| = 2$. [5]

- (c) Show that the transformation $W = \frac{1}{z}$ maps the circle $x^2 + y^2 - 6x = 0$ onto a straight line in W-plane. [4]

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Or

8. (a) If $f(z) = u + iv$ is analytic and $u + v = \sin x \cdot \cosh y + \cos x \cdot \sinh y$, then find $f(z)$ in terms of z . [4]

- (b) Evaluate $\oint_C \frac{e^z}{(z-1)^2(z-2)} dz$ where 'C' is the contour $|z-2| = \frac{3}{2}$
by using Cauchy's residue theorem. [5]

- (c) Find the bilinear transformation which maps the points $0, \frac{1}{2}, 1+i$
from z -plane into the points $-4, \infty, 2-2i$. [4]