

Total No. of Questions—12]

[Total No. of Printed Pages—8+2

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[4162]-201

S.E. (Comp + IT) (First Semester) EXAMINATION, 2012

DISCRETE STRUCTURES

(2008 PATTERN)

Time : Three Hours

Maximum Marks : 100

N.B. :— (i) Answers to the two Sections should be written in separate answer-books.

(ii) Attempt from Section I Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6.

(iii) Attempt from Section II Q. No. 7 or Q. No. 8, Q. No. 9 or Q. No. 10, Q. No. 11 or Q. No. 12.

(iv) Neat diagrams must be drawn wherever necessary.

(v) Figures to the right indicate full marks.

(vi) Assume suitable data, if necessary.

SECTION I

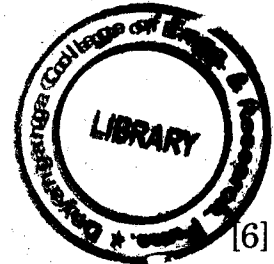
1. (a) Use mathematical induction to show that :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1} \text{ for all } n \geq 1.$$

(b) Consider the following :

p : This system is good

q : This system is cheap



[6]

P.T.O.

Write each of the following sentences in symbolic form : [4]

- (i) This system is good and cheap.
- (ii) This system is not good but cheap.
- (iii) This system is neither good nor cheap.
- (iv) This system is good or cheap.

(c) Obtain the disjunctive normal form of the following : [4]

- (i) $p \vee (\sim p \Rightarrow (q \vee (q \Rightarrow \sim r)))$
- (ii) $p \wedge (p \Rightarrow q)$

(d) Find the power set of : $A = \{ (a, b), c \}$ [2]

Or

2. (a) In a group of 70 cars tested by a garage in a city, 15 had faulty tyres, 20 had faulty brakes and 18 exceed the allowable emission limits. Also, 5 cars had faulty tyres and brakes, 6 failed on tyres and emission, 10 failed on brakes and emission, and 4 cars were unsatisfactory in all three aspects. How many cars had no faults in these three checks ? Draw an appropriate Venn diagram. [6]

- (b) A survey has been taken on modes of travels. Each respondent was asked to check bus, train or automobile as a major modes of travelling for work. More than one answer was permitted. The result, reported outcome were as follows :

Bus-40 people, train-45 people, automobile-100 people, bus and train-20 people, bus and automobile-15 people, train and automobile-20 people and all three modes-5 people. How many people completed a survey form ? Apply principle of inclusion-exclusion. [6]

(c) Let $p(x)$: x is student, $q(x)$: x is clever. Express the following using quantifiers :

(i) There exists a student

(ii) Some students are clever. [2]

(d) Show that $(p \wedge q) \Rightarrow (p \Rightarrow q)$ is a tautology [2]

3. (a) Define the following with example : [6]

(i) Monoid

(ii) Cyclic group

(iii) Abelian group

(b) Show that for any group $(G, *)$, if $a^2 = e$ with $a \neq e$, then G is abelian. [6]

(c) Find the hamming distance between X and Y : [4]

(i) $a(x) = 110110$, $y = 000101$

(ii) $b(x) = 001100$, $y = 010110$

Or

4. (a) Consider the group $(\mathbb{Z}, +)$. Let $H = \{3n : n \in \mathbb{Z}\}$. Show that H is a subgroup of \mathbb{Z} . [6]
- (b) Define a ring. Give examples of the following : [4]
- (i) A commutative ring with identity
 - (ii) A non-commutative ring with identity
 - (iii) Neither commutative nor has a unit element
 - (iv) Integral domain but not a field
- (c) Let 'G' be a group. Show that the function $f : G \rightarrow G$ defined by $f(a) = a^2$ is a homomorphism iff G is abelian. [4]
5. (a) Let A is a set of factors of positive integer m and relation is divisibility on A . i.e. $R = \{(x, y) | x, y \in A, x \text{ divides } y\}$ for $m = 45$, show that POSET (A, \leq) is a lattice. Draw Hasse diagram. Determine the chains and anti-chain. [6]
- (b) Find the transitive closure of R by Warshall's algorithm, [4]
- where $A = \{(1, 3), (3, 1), (2, 4), (4, 2), (3, 5), (5, 3), (4, 6), (6, 4)\}$
- (c) Let $A = \{a, b, c, d\}$, $\pi = \{\{a, b\}, \{c\}, \{d\}\}$. Find the equivalence relation induced by π and construct its digraph. [4]

- (d) Consider the following relation on $\{1, 2, 3, 4, 5, 6\}$: [4]

$$R = \{(i, j) : |i-j| = 2\}$$

Is 'R' transitive ? Is 'R' reflexive ? Is 'R' symmetric ?

Or

6. (a) Using Generating function, solve the following recurrence relation : [8]

$$y_{n+2} - 6y_{n+1} + 8y_n = 0, y_0 = 1, y_1 = 4$$

- (b) Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. In each case state whether the given function (if defined) is injective, surjective, bijective. [4]

(i) $f = \{(1, a), (2, d), (3, b)\}$

(ii) $g = \{(1, a), (2, a), (3, d)\}$

(iii) $h = \{(1, a), (1, b), (2, d), (3, c)\}$ D

(iv) $j = \{(1, a), (2, b)\}$

- (c) Let $A = \{6, 2, 1, 9\}$, $B = \{7, 3, 5\}$, $C = \{8, 4, 10\}$. Obtain the composition of the following functions :

$$f : A \rightarrow B, g : B \rightarrow C$$
 [6]

where $f = \{(6, 7), (2, 3), (1, 5), (9, 3)\}$

$$g = \{(7, 4), (3, 8), (5, 10)\}.$$

SECTION II

7. (a) Define Bipartite graph and Isomorphism graph. Show the isomorphism of two graphs with suitable example. [6]

- (b) Find the shortest path between the vertices a to z in the graph given below by using Dijkstra's algorithm (Fig. 1). [8]

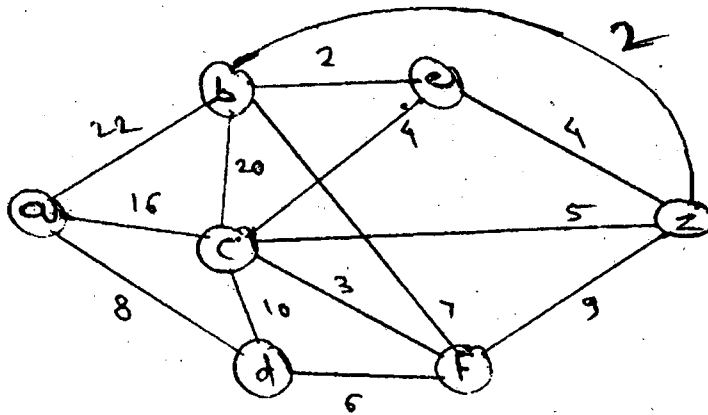


Fig. 1

- (c) Show that a complete graph with n vertices consist of $\frac{n(n+1)}{2}$ edges. [4]

Or

8. (a) Define planar graph. By using Euler's formula, show that if 'G' is connected planar graph with n vertices, e edges and r regions, then : [4]

$$n - e + r = 2$$

- (b) State whether the following statements are true or false. Justify your answer : [6]

- (i) Every tree with $n \geq 2$ vertices is a bipartite graph.
- (ii) A tree is any graph without cycle.
- (iii) Every graph with n vertices and $n - 1$ edges is a tree.
- (iv) If any two vertices of a graph are connected by at least one path, then 'G' is a tree.

- (c) Find all cut-vertices and cut-edges in the graph 'G' shown below (Refer Fig. 2) : [4]

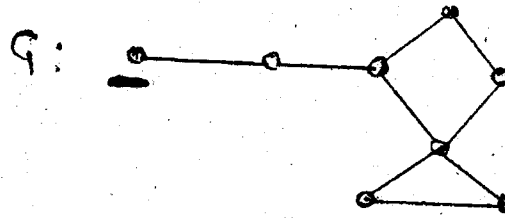


Fig. 2.

- (d) Draw a complete bipartite graph on 2 & 4 vertices $K_{2,4}$ and on 2 & 3 vertices $K_{2,3}$. [4]

9. (a) Define the following terms with suitable example : [6]
- Rooted tree
 - Spanning tree
 - Level of the tree
- (b) Find the minimum cost spanning tree of the following graph using Prim's algorithm. (Refer Fig. 3) [6]

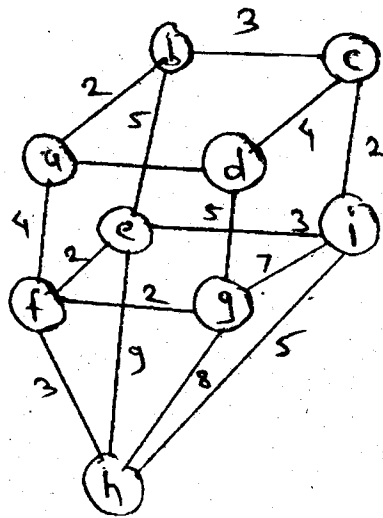


Fig. 3.

- (c) Represent the expression $((a + 5) \uparrow 4) + (b - (5 + 9))$ using a binary tree. [4]

Or

10. (a) Find maximal flow in the transport network using labelling procedure. Determine the corresponding minimum cut. [6]

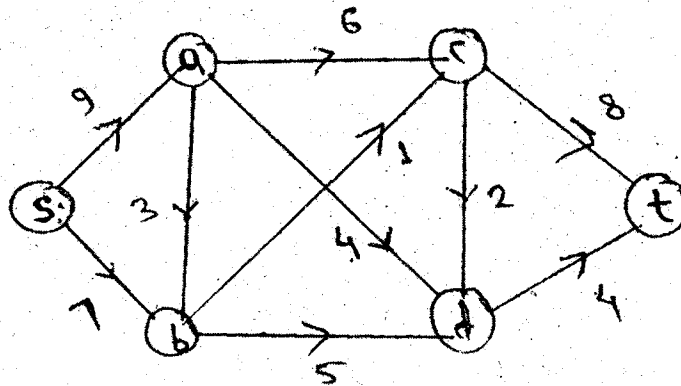


Fig. 4

- (b) Use Kruskal's algorithm to find the minimum spanning tree (MST) of the graph given below (Fig. 5) [6]

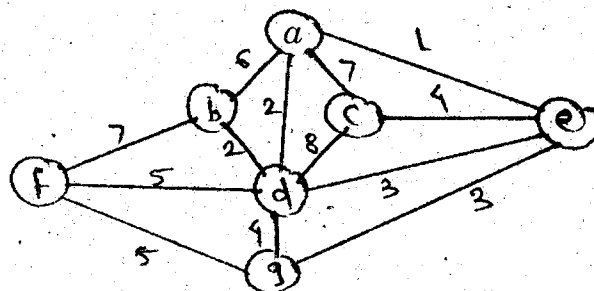


Fig. 5.

- (c) Define optimal prefix code. Obtain an optimal prefix code for the tree of weights 3, 8, 13, 15. [4]

11. (a) If the probability for India to win a game against Australia is 0.4. If India has an option of playing either a "best of three" games or a "best of 5 games" against Australia, which option should India choose so that the probability of winning match is higher ? (Consider both the cases) [6]
- (b) A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or a bolt ? [3]
- (c) In how many ways can seven men and seven women sit down at a round table in such a way that no two men sit next to each other ? [3]
- (d) What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is 1 ? (Assume the probabilities of a 0 and a 1 are the same) [4]

Or

12. (a) An insurance agent sells five policies, one to each of five persons who are of the same age and in good health. According to the actuarial tables the probability that a man of this particular age will be alive 35 years hence is $\frac{2}{3}$. Find the probability that after thirty-five years, at most 3 men will alive. [6]

- (b) In how many ways can the letters of the word ABACUS be arranged such that :
- (i) The vowels always appear together.
 - (ii) Begins with A and end with S. [6]
- (c) Mohan has three shares in a lottery in which there are 3 prizes and 6 blanks. Rohan has one share in a lottery in which there is 1 prize and 2 blanks. Show that Mohan's chance of winning a prize of Rohan's chance is in the ratio 16 : 7. [4]

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