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SEAT No. :

P1237

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M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-701: Functional Analysis

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that quotient space of a Banach Space is a Banach Space. **[6]**

b) Show that a normed linear space N is a Banach Space iff $\{x\in N / \|x\| = 1\}$ is complete. **[5]**

c) Show that set of all continuous linear transformations from Banach Space to itself is a Banach Space. **[5]**

Q2) a) State and prove Hahn-Banach theorem for Banach Spaces. **[6]**

b) Show that if M is a closed linear subspace of Normed linear space N and $x_0 \in N$, $x_0 \notin M$, then $\exists$ a functional f_0 in N*. Such that f_0 (M) = 0 and $f_0(x_0) \neq 0$. **[5]**

c) State and prove Open Mapping theorem for Banach spaces. **[5]**

Q3) a) State and prove Banch-Steinhauss theorem for Banach spaces. **[6]**

b) Show that closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **[5]**

c) Show that for closed linear subspace M of the Hilbert space $M = M^{\perp\perp}$. **[5]**

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Q4) a) State and prove closed graph theorem for Banach spaces. **[6]**

b) Show that if M is a closed linear subspace of Hilbert space H, then $H = M \oplus M^{\perp}$. **[5]**

c) Show that every nonzero Hilbert space contains a complete orthonormal set. **[5]**

Q5) a) Prove that product of self adjoint operators is self adjoint iff they commute. **[6]**

b) State and prove Bessels inequality for Hilbert spaces. **[5]**

c) Show that unitary operators on the Hilbert space forms group under composition. **[5]**

Q6) a) Show that adjoint operator $T \to T^*$ on B(H) has following properties. **[6]**

i) $(T_1T_2)^* = T_2^*T_1^*$

ii) $\|T^*\| = \|T\|$

iii) $(T_1 + T_2)^* = T_1^* + T_2^*$

b) State and prove Schwarz inequality for vectors in Hilbert space. **[5]**

c) Show that an operator T on H is unitary iff it is an isometric isomorphism of H onto itself. **[5]**

Q7) a) Show that if T is normal operator then each eigenspace Mi reduces T. **[6]**

b) State and prove spectral theorem for Hilbert spaces. **[10]**

Q8) a) Show that an operator on Hilbert space is normal iff its real and imaginary part commutes. **[6]**

b) Show that if H is a Hilbert space and f is arbitrary functional on H*, then there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H. **[10]**

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