Total No. of Questions :8]

SEAT No. :

P1415

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[5221] - 204
M.A./M.Sc.
MATHEMATICS
MT-604: Linear Algebra
(2013 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of non - programmable, scientific calculator is allowed.***

Q1) a) Let V be the vector space of all mappings from R to R and V_1, V_2 be the subsets of even and odd functions respectively that is,

$$V_1 = \{f \in V \mid f(-x) = f(x)\}$$

and $V_2 = \{f \in V \mid f(-x) = -f(x)\}$

then show that V is direct sum of V_1 and V_2. **[5]**

b) Find a basis of the subspace of R^4 generated by the vectors $V_1 = (1, 1, 2, 0)$; $V_2 = (1, 2, 3, 4)$; $V_3 = (0, 4, 5, 2)$. **[3]**

c) Find a basis of the vector space $\mathbb{C}$ over $\mathbb{R}$. **[2]**

Q2) a) If V and U are vector spaces over F and $f : V \to U$ is a linear mapping from V onto U, with Kernal K then show that $U \cong V/K$. Further, show that there is a one - to - one correspondence between the set of subspace of V containing K and the set of subspace of U. **[5]**

b) Let $f: R^2 \to R^2$ be a linear mapping, where $f(a, b) = (2a - b, 4a + 5b)$. Find a basis for a range of f and hence determine the rank of f. **[3]**

c) Let $f: R^3 \to R^3$ be defined by $f(x, y, z) = (2x, y - 3z, 1)$. Determine whether f is linear transformation. **[2]**

Q3) a) Let $f: F^{n\times n} \to F^{n\times n}$ be a mapping such that $f(A) = AB$, $A \in F^{n\times n}$ and B is fixed $n\times n$ matrix **[5]**

i) Prove that f is a linear mapping.

ii) Show that $\ker f = (0)$ if and only if B is invertible.

b) Let $f: R^3 \to R^3$ be a linear mapping defined by $f(a, b, c) = (a, a + b, 0)$. Find the matrices A and B respectively of the linear mapping f with respect to the standard basis (e_1, e_2, e_3) and the basis (e_1', e_2', e_3') where $e_1' = (1, 1, 0)$, $e_2' = (0, 1, 1)$, $e_3' = (1, 1, 1)$. **[3]**

c) What is the dimension of the vector space V = {P_n – Polynomial of degree $\le$ n, with real coefficients} **[2]**

Q4) a) If $A \in F^{n\times n}$ matrix has n distinct eigen values $\lambda_1, \lambda_2, ----, \lambda_n$ then show there exists an invertible matrix P such that $P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, ----, \lambda_n)$. **[5]**

b) The three eigen vectors $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, of a 3×3 matrix A are associated respectively with eigen values 1, –1 and 0. Find matrix A. **[3]**

c) Determine the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, if exist. **[2]**

Q5) a) Reduce the following matrix into triangular form, $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -1 & 2 \\ 0 & -3 & 4 \end{bmatrix}$ **[5]**

b) Find the Jordan canonical form of $A = \begin{bmatrix} 1 & 5 & 7 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ **[3]**

c) Determine whether the given set of vectors are orthogonal, S = {(1, 0, 1), (1, 0, 0), (0, –1, 0)} **[2]**

Q6) a) Let V be a vector space of dimension n over F. Then show that there is a 1 – 1 correspondence between the set of bilinear form on V and the set of n×n matrices over F. **[5]**

b) If B is symmetric bilinear form on a vector space V over a field F and let char(F) ≠ 2 then prove that there exists an orthogonal basis of V relative to B. **[3]**

c) If the matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & 4 \\ -2 & 4 & -2 \end{bmatrix}$ then find quadratic form of the matrix A.

[2]

Q7) a) Prove that, if T is a self - adjoint operator on a finite - dimensional Euclidean vector space E then there is an orthonormal basis E consisting of eigen vectors of T. **[5]**

b) Let V be the vector space of continuous real valued functions on the interval [0, 1]. Define $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. Show that < , > is a symmetric bilinear form. **[5]**

Q8) a) State and prove Sylvester's theorem. **[5]**

b) If matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ then find a matrix P such that

$$P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$ **[5]**

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